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TABLES OF SIGNIFICANCE POINTS FOR THE VARIANCE-WEIGHTED KOLMOGO--ETC(U)
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TABLES OF SIGNIFICANCE POINTS FOR THE VARIANCE-WEIGHTED KOLMOGOROV-SMIRNOV STATISTICS

By

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Notations:

$\mathbb{Z}$ stands for the set of all integers,

$\mathbb{R}$ for the real numbers,

$\mathbb{N}_0 := \{n \in \mathbb{Z},\ n \geq 0\}$,

$\mathbb{N}_1 := \{n \in \mathbb{Z},\ n \geq 1\}$,

$x \wedge y := \min\{x,y\}$,

$x \vee y := \max\{x,y\}$,

$(x)_+ := \max\{0,x\}$,

$(x)_- := \min\{0,x\}$,

$\lceil x \rceil := \min\{i \in \mathbb{Z} \mid i \geq x\}$,

$\lfloor x \rfloor := \max\{i \in \mathbb{Z} \mid i \leq x\}$,

$\binom{x}{n} := \dfrac{x(x-1)\cdot\ldots\cdot(x-n+1)}{n!}$ for all $n \in \mathbb{N}_1$; $\binom{x}{0} := 1$; $\binom{x}{z} := 0$ for all $z \notin \mathbb{N}_0$,

For the values of a function $\nu\colon \mathbb{N}_0 \to \mathbb{R}$ we use both notations $\nu(i)$ and ν_i.

Tables of Significance Points for the Variance-Weighted

Kolmogorov-Smirnov Statistics

By

Heinrich Niederhausen

1. Introduction.

Let $X_1,\ldots,X_M$ be i.i.d. random variables with continuous distribution function F and empirical distribution function

$$F_X(x) = M^{-1}\sum_{i=1}^{M} 1_{(-\infty,x]}(X_i) .$$

The goodness-of-fit statistic

$$W_M^+ = \sup_{\theta_1 \le F(x) \le \theta_2} \frac{F_X(x)-F(x)}{\sqrt{F(x)(1-F(x))}}$$

has been shown to be asymptotically minimax (with respect to a certain loss function) by A.A. Borokov and N.M. Sycheva (1968). They also give some exact significance points and the asymptotic distribution of $\sqrt{M}\, W_M^+$. Beside W_M^+, we consider the following related statistics:

$$W_M = \sup_{\theta_1 \le F(x) \le \theta_2} \frac{|F_X(x)-F(x)|}{\sqrt{F(x)(1-F(x))}}$$

$$\tilde{W}_M^+ = \sup_{\theta_1 \le F_X(x) \le \theta_2} \frac{F_X(x)-F(x)}{\sqrt{F(x)(1-F(x))}}$$

$$\tilde{W}_M = \sup_{\theta_1 \le F_X(x) \le \theta_2} \frac{|F_X(x)-F(x)|}{\sqrt{F(x)(1-F(x))}}$$

$$W^+_{M,N} = \sup_{\theta_1 \le F_V(x) \le \theta_2} \frac{|F_X(x)-F_Y(x)|}{\sqrt{F_V(x)(1-F_V(x))}}$$

$$W_{M,N} = \sup_{\theta_1 \le F_V(x) \le \theta_2} \frac{|F_X(x)-F_Y(x)|}{\sqrt{F_V(x)(1-F_V(x))}}$$

$$\tilde{W}^+_{M,N} = \sup_{\theta_1 \le F_X(x) \le \theta_2} \frac{F_X(x)-F_Y(x)}{\sqrt{F_V(x)(1-F_V(x))}}$$

$$\tilde{W}_{M,N} = \sup_{\theta_1 \le F_X(x) \le \theta_2} \frac{|F_X(x)-F_Y(x)|}{\sqrt{F_V(x)(1-F_V(x))}} \quad ,$$

where $Y_1,\ldots,Y_N$ is a second independent sample with the same distribution function, and $V_1,\ldots,V_{M+N}$ is the combined sample. We call all these statistics variance-weighted Kolmogorov-Smirnov tests. In [10], we derived some methods to compute the exact distribution of such tests. Using those methods, we computed tables for the significance points of

$$(1) \qquad \sqrt{M}\, W^+_M,\ \sqrt{M}\, W_M,\ \sqrt{M}\, \tilde{W}^+_M,\ \sqrt{M}\, \tilde{W}_M,\ \sqrt{MN/(M+N)}\, W^+_{M,N},\ \sqrt{MN/(M+N)}\, W_{M,N},$$
$$\sqrt{MN/(M+N)}\, \tilde{W}^+_{M,N} \text{ and } \sqrt{MN/(M+N)}\, \tilde{W}_{M,N} \ .$$

Let Z be any of the eight statistics in (1). Let

$$P(z) = P(Z \le z) \ .$$

For each $\alpha = .9$, $.95$ and $.99$ we try to find z_α such that $P(z_\alpha) = \alpha$. But the variance-weighted Kolmogorov-Smirnov distributions are discontinuous, even in the one-sample case. Therefore, we give $P(\underline{z}_\alpha)$ and $\underline{z}_\alpha$ in the tables, where $P(\underline{z}_\alpha)$ is smaller than α. After each $\underline{z}_\alpha$, a single digit D is printed. If the last digit of $\underline{z}_\alpha$ is increased by D, a $\overline{z}_\alpha$ is obtained, such that $P(\overline{z}_\alpha) \geq \alpha$. $P(\overline{z}_\alpha)$ is also listed. All numbers are rounded in the last digit.

In all the tables we chose $\theta_1 = 1-\theta_2 = \theta$ for $\theta = 0, 0.01, 0.05, 0.1$ and 0.25. In $\tilde{W}_M^+$, $\tilde{W}_M$, $\tilde{W}_{M,N}^+$ and $\tilde{W}_{M,N}$ we have to take the supremum over $\theta \leq F_X(x) \leq 1-\theta$. Thus, we have to replace θ by d/M, where the integer d is chosen such that d/M comes close to θ (see (3.2)). Analogously, replace θ by $d/(M+N)$ in $W_{M,N}^+$ and $W_{M,N}$.

In the two sample case, all tables are given for

$$M = 2,3,4,\ldots,10\ ; \quad N = 2,3,4,\ldots,M$$

$$M = 15,20,25,\ldots,50; \quad N = M,M-1,M-2,\ldots,M-5$$

$$M = 100,500\ ; \ N = M\ .$$

If for small M the table for a certain θ does not differ from the preceding table (with smaller θ), then this part of the table is omitted.

In the one sample, the same values of M are used, but the sample length $M = 500$ is omitted. The computer proved to be too slow for this case (and the desired accuracy).

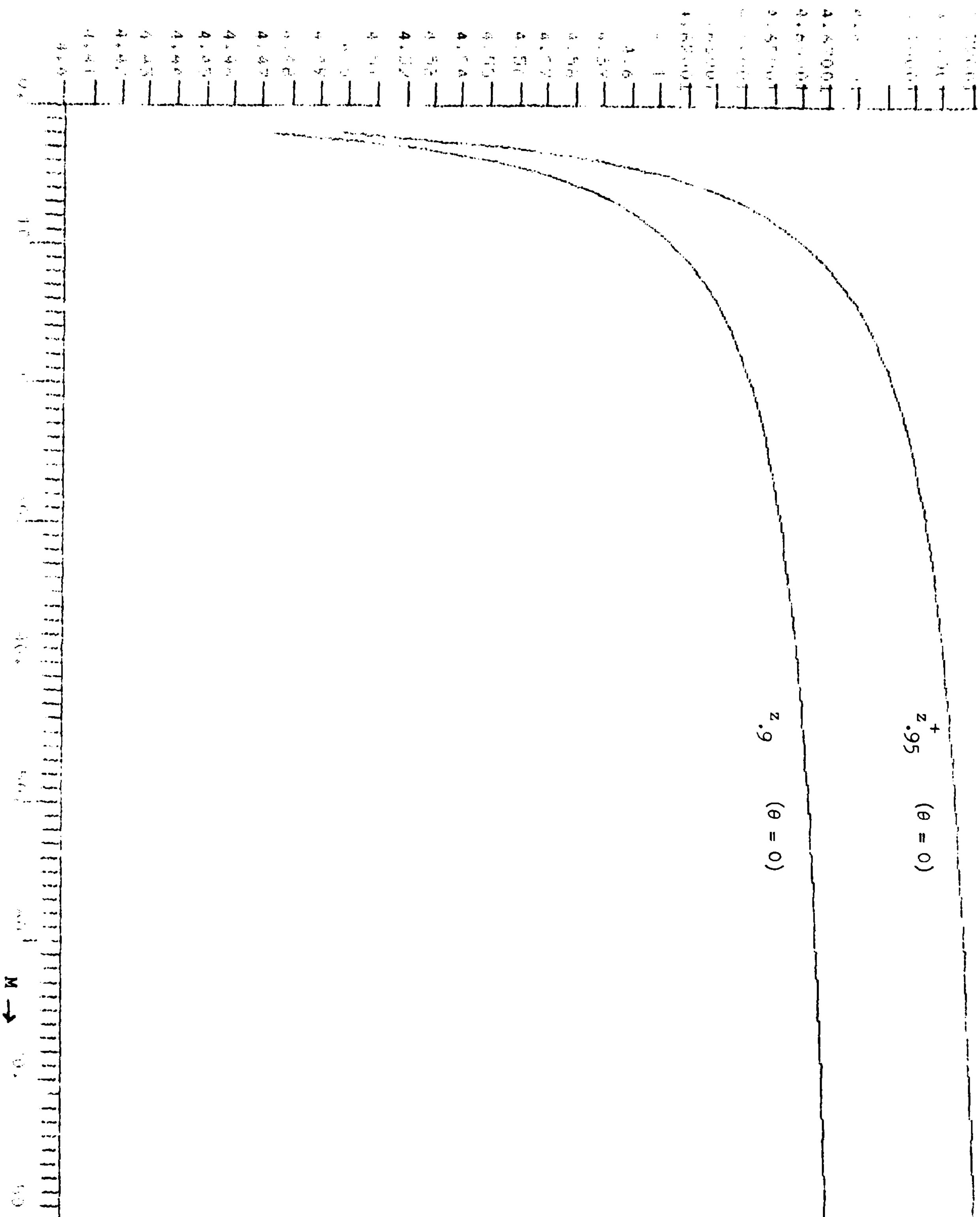
For large sample sizes, a significance value $z_{1-\gamma}$ of a two sided statistic can be approximated by $z^+_{1-\gamma/2}$ of the corresponding one sided statistic. The larger the θ, the better the approximation. Despite "bad" asymptotic behavior, this approximation is practically satisfying even for $\theta = 0$. For this case, the computer drawing on the next page shows $z_{.9}$ and $z^+_{.95}$ for $M = 2,\ldots,80$, where

$$P(\sqrt{M}\, W_M \leq z_{.9})_{\theta=0} \approx .9 \quad\text{and}\quad P(\sqrt{M}\, W^+_M \leq z^+_{.95})_{\theta=0} \approx .95 .$$

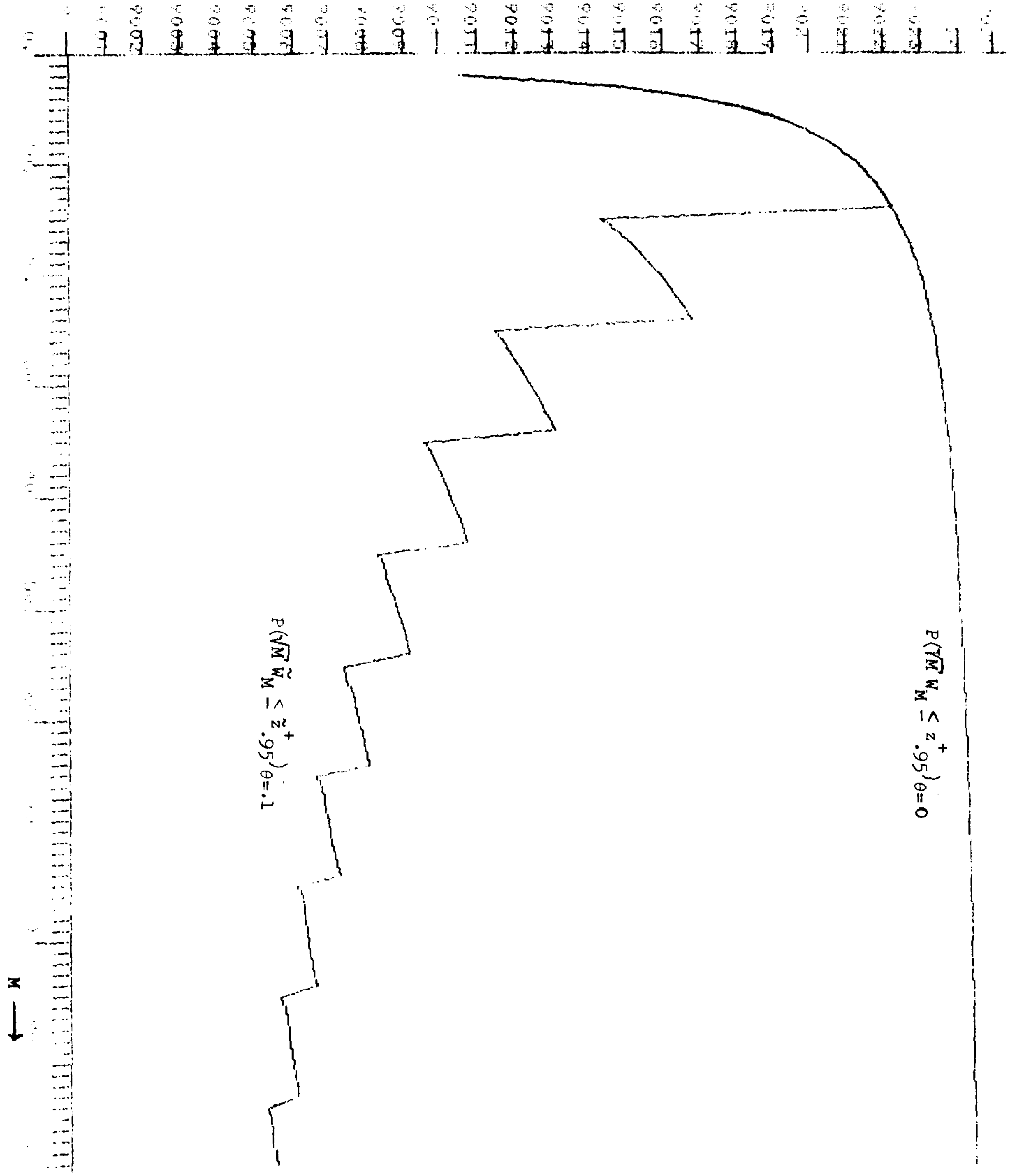
We demonstrate on page 6 what happens if $z^+_{.95}$ (which is much faster to compute) is used as an approximation for $z_{.9}$: We plotted M against $P(\sqrt{M}\, W_M \leq z^+_{.95})_{\theta=0}$ for $M = 2,\ldots,100$. To illustrate the effect of a larger θ, we plotted also $P(\sqrt{M}\, \tilde{W}_M \leq \tilde{z}^+_{.95})_{\theta=1}$. The jumps come from the approximation of θ by d/M.

We repeat that part of [10], which is necessary to understand the algorithms. Chapter 3 gives an overview over the significance points by small tables. The large tables are computed in the same way.

All computations are done on a pdp 11 computer, using 16 significant digits. The reader can compare the tables for $\sqrt{M}\, W^+_M$ with table 1 and 2 of A.A. Borokov and N.M. Sycheva (1968). With the expection of one printing error, their numbers differ at most by 1 in the last given digit. The case $\theta = 0$ for $\sqrt{M}\, W_M$ has been considered by M. Noé (1972). His method of computation is close to ours for all two-sided one sample statistics in (1), except for special cases, see 3.



$z_{.9}$ $(\theta = 0)$
$z^{+}_{.95}$ $(\theta = 0)$
M


P(√M W_M ≤ z+.95)θ=0
P(√M W̃_M ≤ z̃+.95)θ=.1
M

1. One sample tests.

1.1. Sheffer polynomials for D.

Let μ and ν be monotone non-decreasing functions from $\mathbb{N}_0$ into $\mathbb{R}$, satisfying $0 \le \nu_0 \le \mu_0$ and $\nu_i < \mu_{i-1}$ $\forall i \in \mathbb{N}_1$. The following functions define a μ-Sheffer sequence (see (A.12)) for the derivative operator D: $p(x) \mapsto \frac{d}{dx} p(x)$

$$f_0(x) := \begin{cases} 1 & \text{if } x \le 0 \\ 0 & \text{else}, \end{cases}$$

and

$$f_n(x) := \begin{cases} \int_{\nu(n)}^{x \wedge \mu(n-1)} \int_{\nu(n-1)}^{u_n \wedge \mu(n-2)} \cdots \int_{\nu(1)}^{u_2 \wedge \mu(0)} 1 du_1 \ldots du_n & \text{if } x \le \mu_n \\ 0 & \text{else} \end{cases}$$

for all $n \in \mathbb{N}_1$. Obviously, $f_n(\nu_n) = \delta_{0,n}$, hence, (f_n) has roots in ν (see(A.14)).

Denote by $U_{(i)}$ the i-th order statistic of a size M random sample from U(0,1). If $\mu_{M-1} \le 1$, then

$$(1.1) \qquad f_M(\mu_{M-1}) = P(\nu_i \le U_{(i)} \le \mu_{i-1} \ \forall i = 1,\ldots,M)/M! = f_M(\mu_M).$$

1.2. Recursions.

For this section we assume $\mu_M = 1$ without loss of generality. With $q_{n-k}(x) = x^{n-k}/(n-k)!$ in (A.13), the algorithm A.1 was found by M. Nőe (1972). Given that μ_k is constant for $k = 0,\ldots,K$, say, it may be possible to use an explicit formula for $f_i(\mu_K)$ $(i = 0,\ldots,K)$. The same is true for the values p_i in the following application of (A.16): Define $p_0 := 1$ and

$$p_i := \sum_{k=0}^{i-1} \binom{i}{k}(-1)^{i-k-1}(\mu_k - \nu_i)_+^{i-k}\, p_k\,. \tag{1.2}$$

Then

$$P(\nu_i \le U_{(i)} \le \mu_{i-1} \text{ for all } i = 1,\ldots,M) = p_M\,.$$

Alternatively, we get from corollary A.2:

$$p_M = \det\left(\binom{i}{j-1}(\mu_{j-1} - \nu_i)_+^{i-j+1}\right)_{i,j=1,\ldots,M}\,.$$

V.A. Epanechnikov (1968) found recursion (1.2) and G.P. Steck (1971) independently derived this determinant and many applications. See also E.J.G. Pitman (1972) for another proof.

Remark: Depending on the accuracy of the computer, (1.2) should be used only for small M because of the alternating summation. In algorithm A.1 the summation does not alternate, but compared with (1.2) the amount of

computations is approximately squared! In the computation of significance points it often occurs that $\nu_M \le \mu_0$. The same is true, of course, in both one sided cases. Theorem A.2 (with $i := 0$) yields for any real function σ on $\mathbb{N}_0$:

$$(1.3) \qquad p_j = j!t_{j,0}(\sigma_j) - \sum_{k=0}^{j-1} \binom{j}{k}(\sigma_j-\mu_k)^{j-k}p_k \text{ for all } j=1,\ldots,M .$$

From $\nu_M \le \mu_0$ we see that $t_{j,0}$ equals for $j = 0,\ldots,M$ the Sheffer sequence for D with roots in ν. Given that $\sigma_j \ge \mu_{j-1}$ for all $j = 1,\ldots,M$, the summation in (1.3) is non-alternating. But how to compute $t_{j,0}(\sigma_j)$? In the simple case $\nu \equiv 0$ (one sided tests) we get $j!t_{j,0}(\sigma_j) = \sigma_j^j$. With $\sigma_j := \mu_{j-1}$ Steck's formula (1971, (2.3)) is obtained. See the previous section for other closed forms. We suggest the following procedure for general ν (with $\nu_M \le \mu_0$):

Choose $\sigma \equiv 1$. Thus, for all $j = 1,\ldots,M$,

$$j!t_{j,0}(1) = P(\nu_i \le U_{(i)} \text{ for all } i = 1,\ldots,j) = j!f_j^{(j)}(1) ,$$

if $(f_n^{(j)})$ is the $\mu^{(j)}$-Sheffer sequence for D with roots in 0, where $\mu_i^{(j)} = 1-\nu(j-i)$ for all $i = 0,\ldots,j$. Hence, (1.3) can be applied to compute $t_{j,0}(1)$ (we choose again $\sigma \equiv 1$):

$$p_0^{(j)} = 1 \text{ and } p_i^{(j)} = 1 - \sum_{k=0}^{i-1} \binom{i}{k}\nu(j-k)^{i-k}p_k^{(j)} \text{ for all } i=1,\ldots,j .$$

Thus, $j!t_{j,0}(1) = p_j^{(j)}$ for all $j = 1,\ldots,M$. Finally, enter again (1.3) and compute p_M from $p_0 = 1$ and

$$p_j = p_j^{(j)} - \sum_{k=0}^{j-1} \binom{j}{k}(1-\mu_k)^{j-k}p_k \text{ for all } j = 1,\ldots,M .$$

1.3. Rényi type distributions.

In applications, the test distributions seldom occur in the form of (1.1). But if our method is applicable at all, they are easily transformed so that one of the following two lemmas can be used:

Lemma 1.1: Let f and g be monotone non-decreasing functions from $[0,1]$ into itself such that $f < g$ and

$$f(0) \leq a/M < b/M \leq g(1) = 1$$

for two fixed integers a and b. Then

$$P(f(F_U(x)) \leq x \leq g(F_U(x)) \quad \forall a/M \leq F_U(x) \leq b/M)$$
$$= P(\nu_i \leq U_{(i)} \leq \mu_{i-1} \quad \forall i = 1,\dots,M)$$

if

$$\nu_i = \begin{cases} 0 & \text{for all } i = 0,\dots,a-1 \\ f(i/M) & \text{for all } i = a,\dots,b \\ f(b/M) & \text{for all } i > b, \end{cases}$$

and

$$\mu_i = \begin{cases} g(a/M) & \text{for all } i = 0,\dots,a-1 \\ g(i/M) & \text{for all } i = a,\dots,b \\ 1 & \text{for all } i > b\ . \end{cases}$$

The proof is obvious. The situation in the following lemma is much more complicated.

<u>Lemma 1.2</u>: Replace a/M and b/M in lemma 1.1 by any real α and β such that $0 \leq \alpha < \beta \leq 1$. Then, under the same assumptions about f and g,

$$P(f(F_U(x)) \leq x \leq g(F_U(x)) \quad \forall \alpha \leq x \leq \beta)$$

$$= P(\nu_i \leq U_{(i)} \leq \mu_{i-1} \quad \text{for all } i = 1,\dots,M)$$

if

$$\nu_i = \begin{cases} 0 & \text{for all} \quad i = 0,\dots,\alpha_f \\ f(i/M) & \text{for all} \quad i = \alpha_f+1,\dots,\beta_f \\ \beta & \text{for all} \quad i > \beta_f \end{cases}$$

and

$$\mu_i = \begin{cases} \alpha & \text{for all} \quad i = 0,\dots,\alpha_g-1 \\ g(i/M) & \text{for all} \quad i = \alpha_g,\dots,\beta_g-1 \\ 1 & \text{for all} \quad i \geq \beta_g \end{cases}$$

where

$$\alpha_f = \max\{k \leq M \,|\, f(k/M) \leq \alpha\}, \; \beta_f = \max\{k \leq M \,|\, f(k/M) \leq \beta\}$$

$$\alpha_g = \min\{k \geq 0 \,|\, g(k/M) \geq \alpha\}, \; \beta_g = \min\{k \geq 0 \,|\, g(k/M) \geq \beta\}.$$

<u>Proof</u>. Denote by $[0,1]^{(M)}$ the set of all monotone non-decreasing ordered vectors $u \in [0,1]^M$:

$$u = (u_1,\dots,u_M) \text{ such that } 0 \leq u_1 \leq \cdots \leq u_M \leq 1 .$$

Define the subset A of $[0,1]^{(M)}$ by

$A := \{f(i/M) \leq x$ holds for all $i = 0,\dots,M$ and $x \in [u_i, u_{i+1}) \cap [\alpha,\beta]\}$

$(u_0 := 0,\ u_{M+1} := 1)$. Then

$A = \{f(i/M) \leq x$ holds for all $i = \alpha_f+1,\dots,M$ and $x \in [u_i, u_{i+1}) \cap [\alpha,\beta]\}$

$= \{f(i/M) \leq u_i$ holds for all $i = \alpha_f+1,\dots,M$ such that $u_i \leq \beta\}$

$= \{u_{\beta_f+1} > \beta$, and $f(i/M) \leq u_i$ holds for all $i = \alpha_f+1,\dots,\beta_f$ such that $u_i \leq \beta\}$

$= \{u_{\beta_f+1} > \beta$, and $f(i/M) \leq u_i$ holds for all $i = \alpha_f+1,\dots,\beta_f\}$.

By interchanging the roles of f and g it follows analogously that

$B := \{x \leq g(i/M)$ holds for all $i = 0,\dots,M$ and $x \in [u_i, u_{i+1}) \cap [\alpha,\beta]\}$

$= \{u_{\alpha_g} < \alpha$, and $u_i \leq g((i-1)/M)$ for all $i = \alpha_g+1,\dots,\beta_g\}$.

$P(A \cap B) = P(f(F_U(x)) \leq x \leq g(F_U(x))\quad \forall \alpha \leq x \leq \beta)$ finishes the proof. ∎

Remark: If $\nu_{i+1} < \mu_i$ for all $i = 0,\dots,M-1$ in the lemmas above, look for the best applicable method in 1.2. The probability is zero otherwise.

2. Two sample tests.

2.1. Sheffer polynomials for ∇.

Denote by $\mathfrak{T}(i,j)$ the set of all vectors T consisting of exactly i ones and j zeros. For each $T = (T_1,\ldots,T_{i+j}) \in \mathfrak{T}(i,j)$ define the path T'_ℓ of T by $T'_0 := 0$ and $T'_\ell := \Sigma_{k=1}^{\ell} T_k$ for all $\ell = 1,\ldots,i+j$. The set $\mathfrak{T}(i,j)$ is closely related to empirical distribution functions: Let $X_1,\ldots,X_M$, $Y_1,\ldots,Y_N$ be $M+N$ continuous and i.i.d. random variables. Denote the monotone non-decreasing ordered combined sample by $V_1,\ldots,V_{M+N}$. Define a.e.

$$T_\ell = \begin{cases} 1 & \text{if } V_\ell = X_i \text{ for some } i,\ 1 \le i \le M \\ 0 & \text{if } V_\ell = Y_j \text{ for some } j,\ 1 \le j \le N . \end{cases} \tag{2.1}$$

Then $T'_\ell = MF_X(V_\ell)$ and $\ell - T'_\ell = NF_Y(V_\ell)$. Let μ and ν be integer valued function on $\mathbb{N}_0$, $-1 = \nu_0 \le \mu_0$ and

$$\nu_{i-1} - 1 \le \nu_i < \mu_{i-1} \le \mu_i \text{ for all } i \in \mathbb{N}_1 . \tag{2.2}$$

Then $f_i(j) = \# \mathfrak{T}(i,j \,|\, \nu(T'_\ell) < \ell - T'_\ell \le \mu(T'_\ell)$ for all $\ell = 0,\ldots,i+j)$, if (f_n) is the μ-Sheffer sequence (with variables in $\mathbb{Z}$) for the backwards difference operator ∇ (see (A.6)) with roots in ν. Hence,

$$P(\nu(T'_\ell) < \ell - T'_\ell \le \mu(T'_\ell) \text{ for all } \ell = 0,\ldots,M+N) = \binom{M+N}{M}^{-1} f_M(N) . \tag{2.3}$$

2.2. Recursions.

We assume $\mu(M) = N$ throughout this section. From the definition of a μ-Sheffer sequence (f_n) for ∇ with roots in ν we get the following two-dimensional recursion

$$(2.4) \qquad f_i(j) = \begin{cases} f_i(j-1)+f_{i-1}(j) & \text{for all } \nu_i < j \leq \mu_i \\ 0 & \text{else ,} \end{cases}$$

with initial values $f_0(j) = 1$ for all $j \leq \mu_0$, and $f_i(\nu_i) = 0$ for all $i \in \mathbb{N}_1$. On a computer with unlimited integer precision, this algorithm may be slow but absolutely accurate!

The one-dimensional recursion (A.16) is left to the reader. From corollary A.2 one gets the determinantal solution

$$P(\nu(T_\ell') < \ell - T_\ell' \leq \mu(T_\ell') \text{ for all } \ell = 0,\ldots,M+N)$$

$$= \binom{M+N}{N}^{-1} = \det\left(\binom{(\mu_{j-1}-\nu_i)_+}{i-j+1}\right)_{i,j=1,\ldots,n} .$$

This determinant has been found independently by G. Kreweras (1965) and G.P. Steck (1969). See also S.G. Mohanty (1971) and E.J.G. Pitman (1972) for other proofs.

A close look on ν and μ may save some recursion steps. If $\nu(M) < \mu(0)$ the outside method allows non-alternating summation as described in 1.2.

2.3. Rényi type distributions.

Lemma 2.1. Define f, g, a and b as in lemma 1.1. Then

$$P(f(F_X(x)) \leq F_V(x) \leq g(F_X(x)) \text{ for all } a/M \leq F_X(x) \leq b/M)$$

$$= P(\nu(T'_\ell) < \ell - T'_\ell \leq \mu(T'_\ell) \quad \text{for all } \ell = 0,\dots,M+N),$$

if

$$\nu_i = \begin{cases} -1 & \text{for all } i = 0,\dots,a-1 \\ \lceil (M+N)f(i/M)\rceil - i - 1 & \text{for all } i = a,\dots,b \\ \nu_b & \text{for all } i > b\ , \end{cases}$$

and

$$\mu_i = \begin{cases} \mu_a & \text{for all } i = 0,\dots,a-1 \\ \lfloor (M+N)g(i/M)\rfloor - i & \text{for all } i = a,\dots,b \\ N & \text{for all } i > b\ . \end{cases}$$

The proof is obvious from 2.1 .

Lemma 2.2. Define f, g, a and b as in lemma 1.1, and α_f, β_f, α_g and β_g as in lemma 1.2 with $\alpha := a/(M+N)$ and $\beta := b/(M+N)$. Then

$$P(f(F_X(x)) \leq F_V(x) \leq g(F_X(x)) \text{ for all } a/(M+N) \leq F_V(x) \leq b/(M+N))$$

$$= P(\nu(T'_\ell) < \ell \leq \mu(T'_\ell) \qquad \text{for all } \ell = 0,\dots,M+N)\ ,$$

if

$$\nu_i = \begin{cases} -1 & \text{for all } i = 0,\dots,\alpha_f \\ \lceil (M+N)f(i/M)\rceil - i - 1 & \text{for all } i = \alpha_f+1,\dots,\beta_f \\ b-\beta_f-1 & \text{for all } i > \beta_f\ , \end{cases}$$

and

$$\mu_i = \begin{cases} a-\alpha_g & \text{for all } i = 0,\dots,\alpha_g-1 \\ \lfloor (M+N)g(i/M) \rfloor - i & \text{for all } i = \alpha_g,\dots,\beta_g-1 \\ N & \text{for all } i \geq \beta_g \ . \end{cases}$$

The proof follows the same pattern as the proof of lemma 1.2 and is therefore omitted.

Remark: It may happen that ν or μ in lemma 2.1 or 2.2 violates the monotonicity conditions (2.2). In this case define the "monotone hulls" $\tilde{\nu}$ and $\hat{\mu}$ by

$$\text{(2.5)} \qquad \begin{aligned} \tilde{\nu}_0 &:= -1 \\ \tilde{\nu}_i &:= \max\{\nu_i, \tilde{\nu}_{i-1}\} \quad \text{for all } i = 1,\dots,M \ , \end{aligned}$$

and

$$\text{(2.6)} \qquad \begin{aligned} \hat{\mu}_M &:= N \\ \hat{\mu}_i &:= \min\{\mu_i, \hat{\mu}_{i+1}\} \quad \text{for all } i = 0,\dots,M-1 \ . \end{aligned}$$

If $\tilde{\nu}_{i+1} < \hat{\mu}_i$ for all $i = 0,\dots,M-1$, look for the best applicable method in 2.2. The probability is zero otherwise.

3. The variance-weighted Kolmogorv-Smirnov tests.

We defined W_M in the introduction. Let

$$h^{\pm}(i) = \frac{2i+s\pm[s^2+4si(1-i)]^{1/2}}{2(1+s)}$$

and

$$c^{\pm}(\gamma) = M(\gamma \pm[s\gamma(1-\gamma)]^{1/2} .$$

We get from lemma 1.2

$$P(W_M \leq s^{1/2}) = M!f_M(1) ,$$

if (f_n) is the μ-Sheffer sequence for D with roots in ν, where

$$\text{(3.1)} \qquad \nu_i = h^-(i/M) \text{ for all } i = \lfloor c^+(\theta_1)\rfloor \wedge M+1,\ldots,\lfloor c^+(\theta_2)\rfloor \wedge M,$$

and

$$\text{(3.2)} \qquad \mu_i = h^+(i/M) \text{ for all } i = \lceil c^-(\theta_1)\rceil_+,\ldots,\lceil c^-(\theta_2)\rceil_+-1 .$$

The following short tables of the percentage points of $M^{1/2}W_M$ are computed by algoritym A.1 and by the outside method (1.3) if applicable. We chose always $\theta_1 = \theta = 1-\theta_2$ for $\theta = 0$, .01, .05, .1 and .25. Let

$$P(z) = P(M^{1/2}W_M \leq z) .$$

We consider the significance probabilities $\alpha = 1-P(z_\alpha)$ for $\alpha = .1$, $.05$ and $.01$. Because of discontinuities, these levels can not always be attained. If the absolute difference between α and $1-P(z_\alpha)$ is less than $.000005$ this small discontinuity is not noted in the tables, and z_α is rounded to 4 digits after the decimal point. If

$$.000005 \leq |\alpha - 1 + P(z_\alpha)| < .005 \ ,$$

and α is greater(smaller) than $1-P(z_\alpha)$, then five digits are given and a bar is placed under (over) the last digit. This last digit is not rounded. Decreasing (increasing) it by one yields a probability greater (smaller) than α. Two bars indicate an absolute difference between $.005$ and $.013$. The asymptotic values of A.A. Borokov and N.M. Sycheva (1968, Theorem 3A) are given in the last row of table 2-5.

Table 1 is a confirmation of M. Noé's (1972) computations. In table 2 and 3 the results of P.L. Canner's (1975) simulation study are given in parentheses. In table 4 and 5 the rows marked by F contain the percentage points of $M^{1/2}W_M$ as the tables before. The rows marked by F_X refer to the correspondent statistic where the supremum is taken over $d/M \leq F_X(x) \leq 1-d/M$. The integer d is chosen such that d/M is closest to the desired θ:

$$(3.3) \qquad d = \begin{cases} \lfloor M\theta \rfloor & \text{if } M\theta - \lfloor M\theta \rfloor < .5 \\ \lceil M\theta \rceil & \text{else} . \end{cases}$$

The F_X-row in table 4 equals for $M = 10$ the F-row and is therefore omitted.

M	α=.1	α=.05	α=.01
10	4.6146	6.4257	14.1863
20	4.6423	6.4398	14.1908
50	4.6631	6.4488	14.1929
100	4.6719	6.4519	14.1931

Table 1: θ = 0.

M	α=.1	α=.05	α= 01
10	3.2900	3.9829 (3.33)	6.03859 (5.70)
20	3.3962	4.04519 (3.76)	4.9094 (4.77)
50	3.2029	3.55334 (3.69)	4.5353 (4.40)
100	3.0640	3.4379	4.1899
∞	3.05	3.30	3.79

Table 2: θ = .01

M	α=.1	α=.05	α=.01
10	2.9218	3.4216 (3.33)	4.1705 (4.00)
20	2.9094	3.1831 (3.07)	4.10391 (3.80)
50	2.8616	3.1525 (3.15)	3.8289 (3.80)
100	2.8384	3.1417	3.7419
∞	2.89	3.15	3.67

Table 3: θ = .05

M		α=.1	α=.05	α=.01
10	F	2.7148	3.1336	3.8203
20	F	2.7130	2.9830	3.72677
	F_X	3.3938	4.0798	6.1725
50	F	2.7284	3.0071	3.6284
	F_X	2.9641	3.3533	4.2777
100	F	2.7362	3.0120	3.5960
	F_X	2.8562	3.1803	3.8839
∞		2.78	3.05	3.59

Table 4: θ = .1

M		α=.1	α=.05	α=.01
10	F	2.4383	2.6340	3.2863
	F_X	3.2747	3.9777	6.0714
20	F	2.4694	2.7236	3.2852
	F_X	2.7419	3.1507	4.0971
50	F	2.4890	2.77609	3.3414
50	F_X	2.6223	2.9530	3.6498
100	F	2.5159	2.7929	3.3568
	F_X	2.5657	2.8727	3.4969
∞		2.53	2.83	3.40

Table 5: θ = .25

We denote by $W_{M,N}$ the two sample version of W_M:

$$W_{M,N} := \sup_{\theta_1 \le F_V(x) \le \theta_2} \frac{|F_X(x)-F_Y(x)|}{[F_V(x)(1-F_V(x))]^{1/2}}$$

$(\theta_1 = a/(M+N)$ and $\theta_2 = b/(M+N)$; a and b integer) .

Now we get from lemma 2.2

$$P(\frac{N}{M+N} W_{M,N} \le s^{1/2}) = f_M(N)/\binom{M+N}{M} ,$$

if (f_n) is the $\hat{\mu}$-Sheffer sequence for ∇ with roots in $\tilde{\nu}$, where

$$\nu_i = \lceil (M+N)h^-(i/M) \rceil -i-1 \quad \text{and} \quad \mu_i = \lfloor (M+N)h^+(i/M) \rfloor -i ,$$

with i in the same range as in (3.1) and (3.2). (See (2.5) and (2.6) for $\tilde{\nu}$ and $\hat{\mu}$.) The following tables of percentage points for $(\frac{MN}{M+N})^{1/2} W_{M,N}$ are computed using only algorithm (2.4). Discontinuities occur at almost each entry. The bars are set following the same rules as above, but only four digits are given. The table for $\theta = 0$ equals the table for $\theta = .01$ and is therefore omitted. The numbers in parentheses are taken from P.L. Canner's (1975) simulation study (computed for $\theta = 0$). For $\theta = .05$, the rows $M = N = 10$, 20 and 50 are equal to those in table 6, and are omitted. Instead, we demonstrate the effect of slightly different, but large sample sizes. Again, the rows are marked by F_X, if the supremum is taken over all $a'/M \le F_X(x) \le b'/M$. In table 8 and 9 the rows are omitted which do not differ from table 6. The asymptotic values of table 2-5 may be used for comparison.

M=N	$\alpha=.1$	$\alpha=.05$	$\alpha=.01$
10	$2.344\underline{1}$	$2.683\bar{2}$	$3.146\bar{2}$
		(2.71)	(3.17)
20	$2.581\bar{9}$	$2.760\underline{3}$	$3.227\bar{4}$
		(2.70)	(3.18)
50	$2.700\underline{7}$	$2.948\bar{8}$	$3.429\bar{9}$
		(2.93)	(3.44)
100	$2.791\bar{4}$	$3.024\bar{9}$	$3.502\underline{2}$
		(3.02)	(3.47)
500	$2.944\underline{1}$	$3.186\bar{3}$	$3.669\underline{4}$

Table 6: θ = .01

M	N		$\alpha=.1$	$\alpha=.05$	$\alpha=.01$
100	100	F_V	$2.779\bar{5}$	$3.008\underline{9}$	$3.502\underline{2}$
		F_X	$2.736\underline{9}$	$2.982\underline{0}$	$3.472\underline{4}$
100	99	F_V	$2.774\bar{7}$	$3.015\bar{7}$	$3.505\underline{1}$
		F_X	$2.752\underline{4}$	$2.189\bar{3}$	$3.478\bar{1}$
100	98	F_V	$2.760\underline{7}$	$3.014\bar{6}$	$3.511\underline{2}$
		F_X	$2.747\underline{0}$	$2.992\underline{2}$	$3.468\bar{6}$
100	95	F_V	$2.766\bar{9}$	$3.023\bar{9}$	$3.505\bar{7}$
		F_X	$2.742\underline{6}$	$2.993\bar{9}$	$3.486\underline{3}$
500	500	F_V	$2.842\bar{3}$	3.0990	$3.613\underline{6}$

Table 7: θ = .05

M=N	$\alpha=.1$	$\alpha=.05$	$\alpha=.01$
50	$2.666\underline{7}$	$2.896\bar{8}$	$3.429\bar{9}$
100	$2.700\underline{3}$	$2.971\bar{1}$	$3.472\bar{0}$
500	$2.741\bar{5}$	$3.013\underline{9}$	3.5468

Table 8: θ = .1

M=N	$\alpha=.1$	$\alpha=.05$	$\alpha=.01$
20	$2.363\underline{1}$	$2.652\underline{0}$	$3.187\underline{0}$
50	$2.472\underline{3}$	$2.735\bar{7}$	$3.296\bar{3}$
100	$2.482\bar{9}$	$2.758\underline{0}$	$3.313\underline{1}$
500	$2.522\underline{7}$	2.8028	$3.362\underline{3}$

Table 9: θ = .25

For applications see K.A. Doksum and G.L. Sievers (1976): "Plotting with confidence: Graphical comparisons of two populations."

4. Tables

See the introduction for a description of the tables.

$\theta = 0$ / .01 / .05 $\sqrt{M}\, W_M^+$

M	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
						$\theta = 0$						
[illegible]	.9000	[illegible]	[illegible]	8	[illegible]	.9500	4.4975[illegible]	6	[illegible]	.9900	10.021[illegible]	[illegible]
[illegible]	.9000	.9000	3.246598	[illegible]	.9500	.9500	4.556056	5	.9900	.9900	10.04[illegible]	[illegible]
[illegible]	.9000	.9000	3.294[illegible]	[illegible]	.9500	.9500	4.5[illegible]	5	.9900	.9900	10.0602[illegible]	5
[illegible]	.9000	.9000	[illegible]	[illegible]	[illegible]	.9500	4.[illegible]	5	.9900	.9900	10.0688[illegible]	[illegible]
[illegible]	.9000	.9000	[illegible]	7	.9500	.9500	4.[illegible]	5	.9900	.9900	[illegible]	[illegible]
[illegible]	.9000	.9000	[illegible]	[illegible]	.9500	.9500	4.6336[illegible]	5	[illegible]	.9900	[illegible]	[illegible]
[illegible]	.9000	[illegible]	[illegible]	7	.9500	[illegible]	4.6478[illegible]	5	.9900	.9900	10.0812[illegible]	5
[illegible]	.9000	.9000	3.3985[illegible]	[illegible]	.9500	.9500	4.655355	5	.9900	.9900	10.0836[illegible]	5
[illegible]	.9000	.9000	3.40[illegible]	7	.9500	.9500	4.661526	5	.9900	.9900	10.085484	5
[illegible]	.9000	[illegible]	[illegible]	7	.9500	.9500	4.68[illegible]	[illegible]	.9900	.9900	10.0911[illegible]	[illegible]
[illegible]	.9000	.9000	[illegible]	[illegible]	.9500	.9500	4.[illegible]	[illegible]	.9900	[illegible]	10.095[illegible]	5
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	[illegible]	[illegible]	[illegible]	[illegible]	.9900	10.095[illegible]	5
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	.9500	4.[illegible]	5	[illegible]	[illegible]	10.0[illegible]	5
[illegible]	[illegible]	.9000	[illegible]	[illegible]	.9500	.9500	4.708[illegible]	[illegible]	[illegible]	.9900	10.097[illegible]	5
[illegible]	.9000	[illegible]	[illegible]	[illegible]	.9500	.9500	4.[illegible]	5	[illegible]	.9900	10.098[illegible]	5
[illegible]	.9000	.9000	[illegible]	8	.9500	.9500	4.[illegible]	5	[illegible]	.9900	10.09[illegible]	5
[illegible]	.9000	[illegible]	[illegible]	[illegible]	.9500	.9500	4.[illegible]	5	.9900	.9900	10.[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	8	[illegible]	.9500	4[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
						$\theta = .01$						
[illegible]	.9000	.9000	[illegible]	6	[illegible]	.9500	4.4975[illegible]	[illegible]	[illegible]	.9900	[illegible]	6
[illegible]	.9000	.9000	[illegible]	[illegible]	.9500	.9500	4.5560[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	.9000	[illegible]	[illegible]	[illegible]	.9500	.9500	4.589580	5	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	.9000	[illegible]	[illegible]	[illegible]	[illegible]	.9835	4.[illegible]	[illegible]	.9900	.9900	4.[illegible]	5
[illegible]	.9000	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3.8[illegible]	6	[illegible]	.9900	4.[illegible]	[illegible]
[illegible]	.9000	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3.5[illegible]	7	[illegible]	.9900	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	5	[illegible]	.9500	3.2690[illegible]	[illegible]	[illegible]	.9900	[illegible]	5
[illegible]	[illegible]	[illegible]	[illegible]	6	.9500	.9500	3.[illegible]	[illegible]	[illegible]	.9900	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	.9500	3.300705	7	.9900	.9900	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	3.[illegible]0986	6	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	3.[illegible]	[illegible]	[illegible]	.9900	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	6	.9500	.9500	3.[illegible]	[illegible]	.9900	.9900	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	.9500	3.1487[illegible]	6	.9900	.9900	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	3.[illegible]	[illegible]	.9900	.9900	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	3.[illegible]	7	[illegible]	.9913	[illegible]	7
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	.9500	3.2010[illegible]	7	.9900	.9900	[illegible]	[illegible]
[illegible]	.9000	.9000	[illegible]	[illegible]	.9500	.9500	3.[illegible]	[illegible]	.9900	.9900	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
						$\theta = .05$						
[illegible]	[illegible]	[illegible]	[illegible]	6	[illegible]	[illegible]	[illegible]	6	.9900	.9900	[illegible]	5
[illegible]	.9000	.9000	[illegible]	[illegible]	.9500	.9500	[illegible]9086[illegible]4	5	.9900	[illegible]	[illegible]	5
[illegible]	.9000	.9000	[illegible]	[illegible]	.9500	.9500	3.041245	6	[illegible]	[illegible]	[illegible]	8
[illegible]	[illegible]	.9000	[illegible]	5	[illegible]	[illegible]	3.[illegible]	6	.9900	.9900	3.8[illegible]	[illegible]
[illegible]	.9000	[illegible]	[illegible]	[illegible]	.9500	.9500	3.1[illegible]862	[illegible]	.9900	.9900	[illegible]	[illegible]
[illegible]	.9000	[illegible]	[illegible]	5	[illegible]	[illegible]	2.861458	6	.9900	.9900	4.006[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	2.8551[illegible]	9	.9900	.9900	4.06[illegible]	9

θ = .05 / .1 / .25 $\bar{M}' W_M^+$

M	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
					θ = .05 (continued)							
9	[illegible]	[illegible]	2.41[illegible]	[illegible]	[illegible]	[illegible]	2.8[illegible]	[illegible]	.99[illegible]	[illegible]	3.2[illegible]	[illegible]
10	.9000	.9000	[illegible].4501[illegible]	[illegible]	[illegible]	[illegible]	2.[illegible]	6	.9900	.9900	3.[illegible]	[illegible]
15	.9000	.9000	2.551383	5	.9500	.9500	2.64[illegible]	6	.9900	.9934	3.850[illegible]	[illegible]
20	.9000	.9000	2.493085	8	.9500	.9500	2.91[illegible]40	6	.9900	.9900	[illegible]	[illegible]
25	.8979	.9100	2.5[illegible]	[illegible]	.9500	.9500	2.[illegible]	5	.9900	.9900	3.[illegible]	[illegible]
30	.9000	.9000	2.50[illegible]	[illegible]	.9500	.9500	2.8[illegible]1334	6	.9900	.99[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	2.5[illegible]	[illegible]	.9500	.9500	[illegible]87393	5	.9900	.9900	3.[illegible]	6
40	.9000	.9000	2.5058[illegible]	8	.9500	.9500	2.8[illegible]43[illegible]	6	.9909	[illegible]	3.[illegible]	[illegible]
45	.9000	.9000	2.5[illegible]	5	.9500	.9500	2.844615	9	.9900	.9900	3.5600[illegible]	6
50	.9000	.9000	2.508[illegible]	6	.9500	.9500	2.[illegible]417[illegible]	6	.9897	.99[illegible]	3.568[illegible]	[illegible]
100	[illegible]	.9000	2.[illegible]	5	.9500	.9500	2.8[illegible]172	7	.9900	[illegible]	[illegible]	[illegible]
					θ = .1							
[illegible]	[illegible]	[illegible]	[illegible]	9	.9500	.9500	2.635198	5	.9900	1	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	.9500	2.908673	6	.9900	.9900	3.505263	[illegible]
4	.9000	.9000	2.437092	5	.9738	.9600	2.666662	5	.9900	.9900	3.64[illegible]	[illegible]
5	[illegible]	[illegible]55	2.235066	[illegible]	[illegible]	.9500	2.65551[illegible]	[illegible]	.9987	[illegible]	[illegible]	[illegible]
6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.[illegible]3595	5	.9900	.9900	3.41[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	5	[illegible]	[illegible]	2.806885	6	.9900	[illegible]	3.5[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	5	[illegible]	[illegible]	2.624827	5	.9900	[illegible]	[illegible]	[illegible]
[illegible]	.9000	.9000	2.36[illegible]	5	.9500	.9500	2.675036	5	.99[illegible]	.9913	3.444[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	.9500	2.715596	6	.990[illegible]	.9900	3.45[illegible]058	5
[illegible]	.9000	.9000	2.401294	8	.9500	.9500	2.7[illegible]505	5	.9900	[illegible]	3.4[illegible]	[illegible]
[illegible]	.9000	[illegible]	[illegible]	[illegible]	[illegible]	.9500	2.773663	9	.9900	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	2.34[illegible]	5	[illegible]	.9500	2.710359	9	.9900	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	2.35[illegible]	5	.9500	.9500	2.767732	9	.9900	.9900	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	2.300[illegible]	[illegible]	.9500	.9500	2.7[illegible]601[illegible]	9	.9900	[illegible]	3.88[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	2.305[illegible]	8	.9500	.9500	2.708[illegible]1	9	.9900	[illegible]	3.5[illegible]	[illegible]
[illegible]	[illegible]	.9000	[illegible]29[illegible]	8	.9500	.9500	[illegible]732[illegible]	6	.9900	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	.9000	[illegible]97[illegible]	5	.9500	.9530	2.729137	9	.9900	.9900	3.5[illegible]	[illegible]
[illegible]	.9000	.9000	2.400941	5	.9500	.9500	2.73[illegible]851	6	.9900	[illegible]	[illegible]	[illegible]
					θ = .25							
2	.9000	.9000	2.079556	8	.9375	1	2.449487	5	.9375	1	[illegible].4[illegible]9488	[illegible]
3	.9000	.9000	1.880993	7	.9500	.9500	2.[illegible]67822	9	.9844	1	[illegible]	[illegible]
4	.9000	.9000	2.079243	8	.9305	.9663	2.309400	5	.9900	.9900	[illegible]	[illegible]
5	.9000	.9000	1.965651	7	.7500	.9500	2.[illegible]2699	5	.9797	.9918	[illegible]0186	5
6	.9000	.9000	2.060188	8	.9387	.9607	2.[illegible]57020	5	.9900	.9900	3.[illegible]	8
7	.9000	.9000	2.008066	7	.9500	.9500	2.364415	5	.9500	.9900	2.8881[illegible]	[illegible]
8	.9000	.9000	2.088242	8	.9500	.9642	2.449489	5	.9900	.9900	[illegible]	[illegible]
9	.8953	.9103	2.116944	9	.9500	.9500	2.[illegible]754[illegible]0	8	.9900	.9900	2.98[illegible]	[illegible]
10	.9000	.9000	2.089627	7	.9500	.7500	2.438308	5	.9900	.9900	3.032358	[illegible]
15	.9000	.9000	2.[illegible]131[illegible]	9	.9500	.9500	2.464287	5	.9900	.9900	3.08[illegible]	6
20	.9000	.9000	2.122442	9	.9500	.9500	2.469442	5	.9900	.9900	3.092418	7
25	.9000	.9000	2.16219[illegible]	[illegible]	.9500	.9500	2.474890	5	.9900	.9900	3.09[illegible]630	[illegible]
30	.9000	.9000	2.156960	7	.9500	.9500	2.471435	8	.9900	.9900	3.099267	[illegible]
35	.9000	.9000	2.158485	7	.9500	.9500	2.470119	8	.9900	.9900	3.100058	[illegible]
40	.9000	.9000	2.17[illegible]86[illegible]	9	.9500	.9500	2.495123	5	.9900	.9900	3.098[illegible]92	5
45	.9000	.9000	2.171818	7	.9500	.9500	2.493701	8	.9900	.9900	3.098306	5
50	.9000	.9000	2.16592[illegible]	7	.9500	.9500	2.489056	8	.9895	.9901	3.102685	7
100	.9000	.9000	2.185550	5	.9500	.9500	2.515905	5	.9900	.9900	3.1[illegible]8[illegible]	8

θ = 0 / .01 / .05 $\sqrt{M}\, W_M$

M	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
			θ = 0									
2	.9000	.9000	4.473473	9	.9500	.9500	6.333986	6	.9900	.9900	14.149906	7
3	.9000	.9000	4.522930	5	.9500	.9500	6.368907	7	.9900	.9900	14.164893	8
4	.9000	.9000	4.551500	5	.9500	.9500	6.387873	7	.9900	.9900	14.172502	8
5	.9000	.9000	4.570431	5	.9500	.9500	6.399866	7	.9900	.9900	14.177099	8
6	.9000	.9000	4.584036	5	.9500	.9500	6.408166	7	.9900	.9900	14.180173	8
7	.9000	.9000	4.594359	5	.9500	.9500	6.414264	7	.9900	.9900	14.182372	8
8	.9000	.9000	4.602491	5	.9500	.9500	6.418939	7	.9900	.9900	14.184019	8
9	.9000	.9000	4.609091	5	.9500	.9500	6.422640	7	.9900	.9900	14.185293	8
10	.9000	.9000	4.614573	5	.9500	.9500	6.425653	7	.9900	.9900	14.186308	8
15	.9000	.9000	4.632346	5	.9500	.9500	6.434938	7	.9900	.9900	14.189322	7
20	.9000	.9000	4.642249	5	.9500	.9500	6.439753	7	.9900	.9900	14.190768	8
25	.9000	.9000	4.648656	5	.9500	.9500	6.442707	7	.9900	.9900	14.191596	8
30	.9000	.9000	4.653187	5	.9500	.9500	6.444705	7	.9900	.9900	14.192107	8
35	.9000	.9000	4.656583	5	.9500	.9500	6.446151	7	.9900	.9900	14.192440	8
40	.9000	.9000	4.659236	5	.9500	.9500	6.447242	7	.9900	.9900	14.192668	8
45	.9000	.9000	4.661374	5	.9500	.9500	6.448091	7	.9900	.9900	14.192830	8
50	.9000	.9000	4.663134	5	.9500	.9500	6.448777	7	.9900	.9900	14.192936	8
100	.9000	.9000	4.67[illegible]	5	.9500	.9500	6.451909	7	.9900	.9900	14.193053	7

θ = .01

2	.9000	.9000	4.473474	8	.9500	.9500	6.333983	5	.9586	.9969	6.964553	7
3	.9000	.9000	4.522930	5	.9358	.9904	5.628505	6	.9358	.9904	5.628507	7
4	.9000	.9000	4.551499	5	.9113	.9805	4.824180	5	.9900	.9900	5.753819	6
5	.8848	.9670	4.2699[illegible]	8	.8848	.9670	4.269926	9	.9900	.9900	5.862998	6
6	.8564	.9496	3.856864	8	.9500	.9500	3.864508	8	.9900	.9900	5.933793	6
7	.8257	.9282	3.532774	7	.9500	.9500	3.905999	8	.9900	.9900	5.983517	6
8	.7926	.9026	3.[illegible]	6	.9500	.9500	3.937582	8	.9900	.9900	6.020401	6
9	.9000	.9000	3.[illegible]9144	7	.9500	.9500	3.962539	8	.9900	.9900	6.048878	6
10	.9000	.9000	3.289976	7	.9500	.9500	3.982821	9	.9898	.9935	6.038591	5
15	.9000	.9000	3.357[illegible]	6	.9500	.9500	4.046216	9	.9900	.9900	4.853348	5
20	.9000	.9000	3.376235	7	.9483	.9728	4.045195	6	.9900	.9900	4.908577	5
25	.9000	.9000	3.421915	7	.9500	.9500	3.610339	7	.9900	.9900	4.941915	5
30	.9000	.9000	3.139936	6	.9500	.9500	3.634806	8	.9899	.9948	4.954336	5
35	.9000	.9000	3.160673	7	.9500	.9500	3.653253	8	.9829	.9901	4.501876	9
40	.9000	.9000	3.17[illegible]	7	.9500	.9500	3.667791	8	.9900	.9900	4.511206	5
45	.9000	.9000	3.191185	7	.9500	.9500	3.679625	8	.9900	.9900	4.524516	5
50	.9000	.9000	3.202909	7	.9396	.9555	3.553339	6	.9900	.9900	4.535309	5
100	.9000	.9000	3.063968	6	.9500	.9500	3.437878	[illegible]	.9900	.9900	4.189906	7

θ = .05

2	.7708	.9278	2.919985	6	.9500	.9500	3.263289	8	.9900	.9900	5.126814	6
3	.9000	.9000	2.908670	5	.9500	.9500	3.576910	7	.9839	.9973	4.900764	9
4	.9000	.9000	3.038705	6	.9500	.9500	3.721531	7	.9900	.9900	4.196344	9
5	.9000	.9000	3.117920	6	.9385	.9731	3.590922	6	.9900	.9900	4.391910	9
6	.9000	.9000	3.172380	7	.9500	.9500	3.230511	6	.9900	.9900	4.513449	9
7	.8527	.9077	2.861461	6	.9500	.9500	3.298479	7	.9900	.9960	4.595679	5
8	.9000	.9000	2.853165	9	.9500	.9500	3.349445	7	.9834	.9922	4.217750	8
9	.9000	.9000	2.890911	6	.9500	.9500	3.389348	7	.9900	.9900	4.119115	7
10	.9000	.9000	2.9[illegible]	[illegible]	.9500	.9500	3.421598	7	.9900	.9900	4.170525	9

θ = .05 / .1 / .25 $\sqrt{M}\, W_M$

M	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
					θ = .05 (continued)							
15	.9000	.9000	2.840416	6	.9500	.9500	3.265550	6	.9900	.9900	4.011451	8
20	.9000	.9000	2.909351	6	.9500	.9500	3.183144	5	.9900	.9931	4.103908	8
25	.9000	.9000	2.850340	5	.9500	.9500	3.236401	7	.9900	.9900	3.973842	7
30	.9000	.9000	2.888122	6	.9500	.9500	3.175929	5	.9900	.9900	3.886759	6
35	.9000	.9000	2.846932	5	.9500	.9500	3.208063	7	.9900	.9900	3.923989	8
40	.9000	.9000	2.872587	6	.9500	.9500	3.163645	5	.9900	.9900	3.856637	6
45	.9000	.9000	2.842328	9	.9500	.9500	3.186349	7	.9900	.9900	3.881739	8
50	.9000	.9000	2.861627	6	.9500	.9500	3.152505	5	.9900	.9900	3.828912	6
100	.9000	.9000	2.838427	5	.9500	.9500	3.141739	6	.9900	.9900	3.741942	8
					θ = .1							
2	.9000	.9000	2.635194	5	.9500	.9500	3.263297	6	.9800	.9999	4.242639	9
3	.9000	.9000	2.908674	6	.9322	.9790	3.271650	7	.9900	.9900	3.813667	[illegible]
4	.8482	.9201	2.666663	5	.9500	.9500	2.982600	6	.9900	.9900	4.196351	9
5	.9000	.9000	2.655507	9	.9500	.9500	3.134008	6	.9775	.9910	3.726774	8
6	.9000	.9000	2.743020	5	.9500	.9500	3.230508	6	.9900	.9900	3.845438	8
7	.9000	.9000	2.805585	6	.9500	.9500	2.965655	6	.9900	.9900	3.966865	8
8	.9000	.9000	2.624652	5	.9500	.9500	3.036217	6	.9846	.9917	3.771230	7
9	.9000	.9000	2.674590	5	.9500	.9500	3.090404	6	.9900	.9900	3.753606	6
10	.9000	.9000	2.714813	6	.9500	.9500	3.133631	6	.9900	.9900	3.820299	8
15	.9000	.9000	2.716642	5	.9415	.9537	3.012316	5	.9900	.9900	3.795932	6
20	.9000	.9000	2.712983	9	.9500	.9500	2.982984	5	.9893	.9918	3.726779	6
25	.9000	.9000	2.709516	9	.9444	.9516	2.999995	6	.9886	.9907	3.666663	6
30	.9000	.9000	2.706985	9	.9500	.9500	3.037533	6	.9889	.9906	3.651482	6
35	.9000	.9000	2.705311	9	.9500	.9500	3.026680	5	.9897	.9911	3.662328	8
40	.9000	.9000	2.704317	9	.9500	.9500	3.018418	5	.9900	.9900	3.656798	6
45	.9000	.9000	2.732219	6	.9500	.9500	3.012038	5	.9900	.9900	3.641355	6
50	.9000	.9000	2.728351	9	.9500	.9500	3.007049	5	.9900	.9900	3.628393	6
100	.9000	.9000	2.736163	5	.9500	.9500	3.012031	5	.9900	.9900	3.595980	6
					θ = .25							
2	.8750	.9999	2.449489	5	.8750	.9999	2.449486	6	.8750	.9999	2.449488	5
3	.9000	.9000	2.267873	9	.9500	.9500	2.694409	5	.9687	.9999	2.999998	6
4	.8611	.9325	2.309397	5	.9500	.9500	2.461594	5	.9900	.9900	3.323007	7
5	.9000	.9000	2.342701	5	.9500	.9500	2.722945	5	.9900	.9900	3.070340	6
6	.8773	.9213	2.357022	5	.9500	.9500	2.576874	9	.9884	.9960	3.299829	7
7	.9000	.9000	2.364412	5	.9500	.9500	2.715605	5	.9900	.9900	3.197025	5
8	.8999	.9285	2.449485	5	.9500	.9500	2.615492	9	.9877	.9937	3.265986	7
9	.9000	.9000	2.375411	8	.9500	.9500	2.707557	5	.9900	.9900	3.233146	5
10	.9000	.9000	2.438304	5	.9500	.9500	2.633963	9	.9886	.9931	3.286328	7
15	.9000	.9000	2.464260	5	.9500	.9500	2.696417	5	.9900	.9900	3.249258	6
20	.9000	.9000	2.469403	5	.9500	.9500	2.723559	6	.9900	.9900	3.285203	7
25	.9000	.9000	2.474835	5	.9500	.9500	2.732763	6	.9900	.9900	3.301083	7
30	.9000	.9000	2.471383	8	.9500	.9500	2.742865	6	.9900	.9900	3.309647	7
35	.9000	.9000	2.470067	8	.9500	.9500	2.772704	6	.9900	.9900	3.313743	7
40	.9000	.9000	2.495068	5	.9500	.9500	2.769853	9	.9900	.9900	3.317094	7
45	.9000	.9000	2.493637	8	.9500	.9500	2.768674	9	.9900	.9900	3.343975	7
50	.9000	.9000	2.489001	8	.9484	.9509	2.776085	6	.9900	.9900	3.341443	5
100	.9000	.9000	2.515856	5	.9500	.9500	2.792941	5	.9900	.9900	3.356768	6

$\theta = 0$ / .01 / .05 $\sqrt{M}\,\tilde{W}_M^+$

For $\theta = 0$ see $\sqrt{M}\,W_M^+$

$\theta = .01$

M	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
2	.9000	.9000	3.168531	8	.9500	.9500	4.497598	6	.9900	.9900	10.021762	9
3	.9000	.9000	3.246598	7	.9500	.9500	4.556056	5	.9900	.9900	10.042498	5
4	.9000	.9000	3.294047	7	.9500	.9500	4.589382	5	.9900	.9900	10.060775	5
5	.9000	.9000	3.326808	7	.9500	.9500	4.611263	[illegible]	.9900	.9900	10.068689	5
6	.9000	.9000	3.351192	7	.9500	.9500	4.626872	5	.9900	.9900	10.0743[illegible]	5
7	.9000	.9000	3.370260	7	.9500	.9500	4.638649	5	.9900	.9900	10.078309	5
8	.9000	.9000	3.385706	7	.9500	.9500	4.647888	5	.9900	.9900	10.081[illegible]	5
9	.9000	.9000	3.398569	7	.9500	.9500	4.65535[illegible]	5	.9900	.9900	10.0[illegible]614	5
[illegible]	.9000	.9000	3.[illegible]09468	7	.9500	.9500	4.6615[illegible]	5	[illegible]	.9900	[illegible]5464	5
[illegible]	.9000	.9000	3.446899	7	.9500	.9500	4.6[illegible]140[illegible]	[illegible]	[illegible]	.9900	[illegible]	9
[illegible]	[illegible]	.9000	3.4[illegible]	[illegible]	.9500	.9500	4.69[illegible]35[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	.9000	3.485[illegible]	7	.9500	.9500	4.6[illegible]	5	[illegible]	[illegible]	10.[illegible]	[illegible]
[illegible]	[illegible]	.9000	3.496965	[illegible]	.9500	.9500	4.[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	3.[illegible]	[illegible]	.9500	.9500	4.[illegible]	5	[illegible]	.9900	10.[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	[illegible]	4.[illegible]	[illegible]	[illegible]	.9900	[illegible]	[illegible]
4[illegible]	[illegible]	[illegible]	3.[illegible]048	[illegible]	.950[illegible]	[illegible]	4.[illegible]	[illegible]	[illegible]	[illegible]	10.[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	8	.9500	.9500	4.[illegible]	5	[illegible]	.9900	10.0[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	5	.9500	.9500	4.[illegible]435[illegible]	[illegible]	.9900	[illegible]	10.[illegible]	[illegible]

$\theta = .05$

M	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	.9500	4.4[illegible]599	8	[illegible]	.9900	10.[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	7	.9500	.9500	4.[illegible]054	5	[illegible]	.9900	10.[illegible]	[illegible]
[illegible]	[illegible]	.9000	3.[illegible]	[illegible]	.9500	[illegible]	4.[illegible]	5	[illegible]	.9900	10.[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	3.[illegible]	[illegible]	.9500	.9500	4.611261	5	[illegible]	.9900	10.[illegible]	5
[illegible]	[illegible]	.9000	[illegible]351182	[illegible]	.9500	.9500	4.[illegible]	5	.9900	.9900	10.[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	3.370[illegible]	[illegible]	.9500	.9500	4.6[illegible]	[illegible]	.9900	.9900	10.078[illegible]	[illegible]
[illegible]	[illegible]	.9000	3.385[illegible]	[illegible]	.9500	.9500	4.6[illegible]	[illegible]	[illegible]	.9900	10.081[illegible]	[illegible]
[illegible]	[illegible]	.9000	3.398[illegible]	[illegible]	.9500	.9500	4.65[illegible]	[illegible]	[illegible]	.9900	10.085[illegible]	[illegible]
[illegible]	.9000	.9000	3.[illegible]	[illegible]	.9500	.9500	4.6[illegible]35[illegible]	[illegible]	.9900	.9900	10.08548[illegible]	5
[illegible]	[illegible]	.90[illegible]	3.44[illegible]	[illegible]	.9500	.9500	4.6[illegible]	8	[illegible]	.9900	10.[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	.9500	4.69[illegible]	[illegible]	.9900	.9900	10.[illegible]	[illegible]
[illegible]	.9000	[illegible]	[illegible]	[illegible]	.9500	.9500	4.[illegible]	[illegible]	[illegible]	.9900	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	.9500	3.4[illegible]	6	.9900	.9900	[illegible]	5
[illegible]	[illegible]	.9000	[illegible]	6	.9500	.9500	3.4[illegible]1040	7	.9900	.9900	5.[illegible]	[illegible]
[illegible]	[illegible]	.9000	[illegible]	6	.9500	.9500	3.4[illegible]	[illegible]	.9900	.9900	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	6	.9500	.9500	3.4[illegible]	7	.9900	.9900	[illegible]	[illegible]
[illegible]	.9000	.9000	[illegible]	5	.9500	.9500	3.[illegible]	6	.9900	.9900	[illegible]	[illegible]
[illegible]	.9000	.9000	[illegible]	[illegible]	.9500	.9500	3.0[illegible]399	[illegible]	.9900	.9900	3.[illegible]4408[illegible]	[illegible]

θ = .1 / .25 $\sqrt{M}\,\tilde{W}_M^+$

θ = .1

M	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
2	.9000	.9000	3.168534	6	.9500	.9500	4.497596	8	.9900	.9900	10.021760	8
3	.9000	.9000	3.246599	7	.9500	.9500	4.556057	5	.9900	.9900	10.047475	[illegible]
4	.9000	.9000	3.294048	7	.9500	.9500	4.589383	5	.9900	.9900	10.0[illegible]0775	5
5	.9000	.9000	3.308034	7	.9500	.9500	4.606667	5	.9900	.9900	10.068841	[illegible]
6	.9000	.9000	3.340215	7	.9500	.9500	4.624908	5	.9900	.9900	10.074[illegible]	5
7	.9000	.9000	3.363255	7	.9500	.9500	4.637705	5	.9900	.9900	10.078307	5
8	.9000	.9000	3.380929	7	.9500	.9500	4.64[illegible]391	5	.9900	.9900	10.081788	[illegible]
9	.9000	.9000	3.395137	7	.9500	.9500	4.655070	5	.9900	.9900	10.083616	5
10	.9000	.9000	3.406929	7	.9500	.9500	4.661357	5	.9900	.9900	10.08548[illegible]	5
15	.9000	.9000	2.755779	6	.9500	.9500	3.366268	7	.9900	.9900	[illegible]	[illegible]
20	.9000	.9000	2.808701	6	.9500	.9500	3.408647	7	.9900	.9900	[illegible]	[illegible]
25	.9000	.9000	2.631816	5	.9500	.9500	3.113591	6	.9900	.9900	[illegible]	[illegible]
30	.9000	.9000	2.666876	5	.9500	.9500	3.143281	[illegible]	.9900	.9900	[illegible]	[illegible]
35	.9000	.9000	2.578749	9	.9500	.9500	3.008210	5	.9900	.9900	[illegible]	[illegible]
40	.9000	.9000	2.604833	5	.9500	.9500	3.030824	6	.9900	.9900	[illegible]	[illegible]
45	.9000	.9000	2.547112	9	.9500	.9500	2.956783	5	.9900	.9900	3.858856	[illegible]
50	.9000	.9000	2.579438	5	.9500	.9500	2.969184	6	.9900	.9900	[illegible]	[illegible]
100	.9000	.9000	2.508909	6	.9500	.9500	2.858875	7	.9900	.9900	[illegible]	[illegible]

θ = .25

M	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
2	.9000	.9000	3.168537	6	.9500	.9500	4.497594	8	.9900	.9900	10.021762	8
3	.9000	.9000	3.160504	7	.9500	.9500	4.51[illegible]855	5	.9900	.9900	[illegible]	5
4	.9000	.9000	3.257602	7	.9500	.9500	4.576840	5	.9900	.9900	10.[illegible]0309	5
5	.9000	.9000	3.308036	7	.9500	.9500	4.606666	5	.9900	.9900	10.068843	5
6	.9000	.9000	3.340214	7	.9500	.9500	4.624908	5	.9900	.9900	10.074561	5
7	.9000	.9000	2.529286	7	.9500	.9500	3.176185	6	.9900	.9900	[illegible]	5
8	.9000	.9000	2.584677	5	.9500	.9500	3.223949	7	.9900	.9900	5.027019	5
9	.9000	.9000	2.625684	5	.9500	.9500	3.258906	7	.9900	.9900	5.0552[illegible]	5
10	.9000	.9000	2.658176	5	.9500	.9500	3.265989	7	.9900	.9900	5.077296	5
15	.9000	.9000	2.328809	9	.9500	.9500	2.786214	5	.9900	.9900	3.837155	[illegible]
20	.9000	.9000	2.316607	8	.9500	.9500	2.744[illegible]	9	.9900	.9900	[illegible]	[illegible]
25	.9000	.9000	2.311667	7	.9500	.9500	2.717556	9	.9900	.9900	[illegible]	[illegible]
30	.9000	.9000	2.306731	7	.9500	.9500	2.699178	9	.9900	.9900	[illegible]	6
35	.9000	.9000	2.246285	8	.9500	.9500	2.626054	9	.9900	.9900	3.4087[illegible]	6
40	.9000	.9000	2.251801	5	.9500	.9500	2.625135	9	.9900	.9900	3.368390	5
45	.9000	.9000	2.255946	5	.9500	.9500	2.624134	9	.9900	.9900	3.371874	5
50	.9000	.9000	2.259191	5	.9500	.9500	2.623170	9	.9900	.9900	3.358227	5
100	.8999	.9000	2.220834	5	.9500	.9500	2.566108	6	.9900	.9900	3.239695	[illegible]

$\theta = 0$ / .01 / .05 $\quad \widetilde{M}\ \tilde{W}_M$

For $\theta = 0$ see $\widetilde{M}\ W_M$

$\theta = .01$

M	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
[illegible]	[illegible]	.9000	[illegible]	9	.9500	.9500	[illegible]	6	.9900	.9900	[illegible]	7
[illegible]	[illegible]	.9000	[illegible]	5	.9500	.9500	6.368915	7	.9900	.9900	[illegible]	8
4	.9000	[illegible]	4.551500	5	.9500	.9500	[illegible]	[illegible]	.9900	.9900	[illegible]	[illegible]
5	[illegible]	.9000	4.570436	5	.9500	.9500	6.399873	7	.9900	.9900	[illegible]	[illegible]
6	.9000	[illegible]	4.584041	5	.9500	.9500	6.408173	7	.9900	.9900	[illegible]	[illegible]
[illegible]	.9000	.9000	[illegible]	5	.9500	.9500	[illegible]	7	.9900	.9900	[illegible]	8
[illegible]	.9000	.9000	[illegible]	5	.9500	.9500	6.418953	[illegible]	.9900	.9900	[illegible]	8
[illegible]	.9000	.9000	[illegible]	5	.9500	.9500	6.422654	7	.9900	.9900	[illegible]	8
[illegible]	.9000	.9000	[illegible]	5	.9500	.9500	6.425667	7	.9900	.9900	[illegible]	[illegible]
[illegible]	.9000	[illegible]	[illegible]	5	.9500	.9500	[illegible]	[illegible]	.9900	.9900	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	5	.9500	[illegible]	[illegible]	[illegible]	.9900	.9900	[illegible]	8
[illegible]	[illegible]	[illegible]	[illegible]	5	[illegible]	.9500	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	8
[illegible]	[illegible]	.9000	[illegible]	5	.9500	.9500	[illegible]	7	.9900	.9900	[illegible]	8
[illegible]	[illegible]	[illegible]	[illegible]	5	.9500	.9500	6.446180	7	.9900	.9900	[illegible]	8
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	[illegible]	7	[illegible]	.9900	[illegible]	8
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	.9500	6.448135	7	[illegible]	.9900	[illegible]	8
[illegible]	.9000	[illegible]	[illegible]	5	.9500	.9500	6.448821	7	.9900	.9900	[illegible]	8
[illegible]	.9000	.9000	[illegible]	6	.9500	.9500	6.452548	9	.9900	.9900	[illegible]	9

$\theta = .05$

M	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
[illegible]	.9000	.9000	[illegible]	8	.9500	.9500	[illegible]	5	[illegible]	.9900	[illegible]	6
[illegible]	[illegible]	[illegible]	[illegible]	5	.9500	.9500	6.368914	7	.9900	.9900	[illegible]	8
[illegible]	[illegible]	[illegible]	[illegible]	5	[illegible]	.9500	6.387880	7	[illegible]	.9900	[illegible]	[illegible]
5	[illegible]	.9000	[illegible]	5	.9500	.9500	6.399873	7	.9900	.9900	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	.9500	[illegible]	[illegible]	.9900	.9900	[illegible]	[illegible]
[illegible]	[illegible]	.9000	[illegible]	5	.9500	.9500	[illegible]	7	[illegible]	.9900	[illegible]	8
[illegible]	[illegible]	[illegible]	[illegible]	5	.9500	.9500	[illegible]	7	[illegible]	.9900	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	.9500	[illegible]	7	.9900	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	5	.9500	.9500	6.425660	7	.9900	.9900	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.9500	.9500	6.434961	7	[illegible]	.9900	[illegible]	7
[illegible]	[illegible]	[illegible]	[illegible]	5	.9500	.9500	6.439782	7	[illegible]	.9900	[illegible]	8
[illegible]	[illegible]	[illegible]	[illegible]	5	.9500	.9500	6.442737	7	.9900	.9900	[illegible]	8
[illegible]	[illegible]	.9000	[illegible]	5	.9500	.9500	[illegible]	9	.9900	.9900	[illegible]	8
[illegible]	.9000	[illegible]	[illegible]	7	.9500	.9500	[illegible]	9	.9900	.9900	[illegible]	7
[illegible]	[illegible]	[illegible]	[illegible]	7	.9500	.9500	[illegible]	9	.9900	.9900	[illegible]	[illegible]
[illegible]	[illegible]	.9000	[illegible]	[illegible]	.9500	.9500	4.143886	9	.9900	.9900	[illegible]	[illegible]
[illegible]	.9000	[illegible]	[illegible]	6	.9500	.9500	3.689012	7	.9900	.9900	[illegible]	6
[illegible]	[illegible]	[illegible]	[illegible]	8	.9500	.9500	[illegible]	[illegible]	.9900	.9900	[illegible]	[illegible]

θ = .1 / .25 $\sqrt{M}\,\tilde{W}_M$

θ = .1

M	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
2	.9000	.9000	4.473473	8	.9500	.9500	6.333989	5	.9900	.9900	14.14995[illegible]	6
3	.9000	.9000	4.522929	5	.9500	.9500	6.368911	7	.9900	.9900	14.16496[illegible]	7
4	.9000	.9000	4.551504	5	.9500	.9500	6.387877	7	.9900	.9900	14.172566	8
5	.9000	.9000	4.565463	5	.9500	.9500	6.399059	7	.9900	.9900	14.177200	[illegible]
6	.9000	.9000	4.581875	5	.9500	.9500	6.407950	7	.9900	.9900	14.18029[illegible]	[illegible]
7	.9000	.9000	4.593308	5	.9500	.9500	6.414203	7	.9900	.9900	14.182[illegible]	[illegible]
8	.9000	.9000	4.601931	5	.9500	.9500	6.418928	7	.9900	.9900	14.184[illegible]	[illegible]
9	.9000	.9000	4.608778	5	.9500	.9500	6.422644	7	.9900	.9700	14.18547[illegible]	[illegible]
10	.9000	.9000	4.614381	5	.9500	.9500	6.425664	7	.9900	.9900	14.18651[illegible]	[illegible]
15	.9000	.9000	3.352703	6	.9500	.9500	4.045169	8	.9900	.9900	[illegible]	6
20	.9000	.9000	3.393782	7	.9500	.9500	4.079796	9	.9900	.9900	[illegible]	[illegible]
25	.9000	.9000	3.105391	6	.9500	.9500	3.608057	7	.9900	.9900	4.941[illegible]	6
30	.9000	.9000	3.134453	7	.9500	.9500	3.633362	8	.9900	.9900	[illegible]	5
35	.9000	.9000	3.002223	5	.9500	.9500	3.431598	6	.9900	.9900	4.4[illegible]	[illegible]
40	.9000	.9000	3.024452	6	.9500	.9500	3.451278	7	.9900	.9900	4.5[illegible]	5
45	.9000	.9000	2.946202	5	.9500	.9500	3.337[illegible]06	6	.9900	.9900	4.26[illegible]3	8
50	.9000	.9000	2.964130	6	.9500	.9500	3.353256	7	.9900	.9900	4.[illegible]	[illegible]
100	.9000	.9000	2.8[illegible]	7	.9500	.9500	3.18[illegible]	8	.99[illegible]	[illegible]	3.88[illegible]	[illegible]

θ = .25

M	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
2	.9000	.9000	4.423475	8	.9500	.9500	6.333989	5	.9900	.9900	14.149953	6
3	.9000	.9000	4.477702	5	.9500	.9500	6.349215	7	.9900	.9900	14.162796	8
4	.9000	.9000	4.538141	5	.9500	.9500	6.384351	7	.9700	.9900	14.17249[illegible]	[illegible]
5	.9000	.9000	4.565459	5	.9500	.9500	6.399056	7	.9900	.9900	14.177196	8
6	.9000	.9000	4.581877	5	.9500	.9500	6.407948	7	.9900	.9900	14.180295	8
7	.9000	.9000	3.162[illegible]04	6	.9500	.9500	3.883435	9	.9900	.99[illegible]	[illegible]80360	8
8	.9000	.9000	3.214065	7	.9500	.9500	3.924698	8	.9900	.9900	[illegible]019281	6
9	.9000	.9000	3.245[illegible]	7	.9500	.9500	3.954623	8	.9900	.9900	[illegible]045438	6
10	.9000	.9000	3.274686	7	.9500	.9500	3.977673	8	.9900	.9900	6.071347	6
15	.9000	.9000	2.783164	5	.9500	.9500	3.232701	5	.9900	.9900	4.314027	9
20	.9000	.9000	2.741885	9	.9500	.9500	3.150680	5	.9900	.9900	4.097055	[illegible]
25	.9000	.9000	2.715561	9	.9500	.9500	3.099434	5	.9900	.9900	3.965390	7
30	.9000	.9000	2.697479	9	.9500	.9500	3.064553	5	.9900	.9900	3.827288	6
35	.9000	.9000	2.625000	9	.9500	.9500	2.974293	5	.9900	.9900	3.[illegible]26047	6
40	.9000	.9000	2.624162	9	.9500	.9500	2.965851	5	.9900	.9900	3.695134	6
45	.9000	.9000	2.623225	9	.9500	.9500	2.958838	5	.9900	.9900	3.670236	6
50	.9000	.9000	2.622306	9	.9500	.9500	2.952950	5	.9900	.9900	3.649772	6
100	.9000	.9000	2.565736	6	.9500	.9500	2.872725	6	.9900	.9900	3.496974	8

$\theta = 0$ $\sqrt{MN/(M+N)}\, W^{+}_{M,N}$

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
2	2	.8333	1	1.999992	7	.8333	1	1.999992	8	.8333	1	1.999992	8
3	3	.7000	.9500	1.732050	6	.7000	.9500	1.732050	7	.9500	1	2.449488	5
3	2	.7000	.9000	1.490711	9	.9000	1	2.236060	8	.9000	1	2.236060	9
4	4	.8857	.9857	2.190888	9	.8857	.9857	2.190888	9	.9857	1	2.828421	7
4	3	.8857	.9714	1.984307	8	.8857	.9714	1.984307	8	.9714	1	2.645751	6
4	2	.8000	.9333	1.732050	9	.9333	1	2.449489	5	.9333	1	2.449489	5
5	5	.8571	.9603	2.070193	9	.8571	.9603	2.070193	8	.9603	.9960	2.581982	7
5	4	.8571	.9286	1.897360	7	.9286	.9603	2.371704	6	.9603	.9921	2.399999	5
5	3	.8571	.9286	2.108180	6	.9286	.9821	2.190888	8	.9821	1	2.828420	7
5	2	.8571	.9524	1.932181	5	.8571	.9524	1.932181	8	.9524	1	2.645748	6
6	6	.8301	.9286	1.999997	9	.9459	.9870	2.449489	7	.9870	.9989	2.927692	8
6	5	.8810	.9069	2.100524	8	.9459	.9762	2.288686	8	.9870	.9978	2.763848	8
6	4	.8238	.9095	1.936486	8	.9095	.9524	2.108183	7	.9762	.9952	2.581988	7
6	3	.8929	.9524	2.267783	6	.8929	.9524	2.267783	9	.9881	1	2.999999	7
6	2	.8929	.9643	2.108183	5	.8929	.9643	2.108183	9	.9643	1	2.828425	6
7	7	.8939	1	2.160237	8	.9312	.9735	2.366425	8	.9808	.9959	2.788864	8
7	6	.8858	.9266	2.133073	7	.9266	.9545	2.225391	8	.9808	.9924	2.638986	8
7	5	.8813	.9167	2.070194	7	.9356	.9672	2.366429	7	.9848	.9924	2.898274	8
7	4	.8636	.9394	2.068275	8	.9394	.9667	2.288686	8	.9848	.9970	2.746421	8
7	3	.8750	.9167	2.070190	8	.9167	.9667	2.415228	7	.9667	.9917	2.535456	9
7	2	.7500	.9167	1.984307	8	.9167	.9722	2.267783	7	.9722	1	2.999998	7
8	8	.8738	.9148	2.065584	8	.9148	.9565	2.309395	7	.9740	.9907	2.696797	8
8	7	.8651	.9037	2.070191	7	.9417	.9557	2.351447	7	.9883	.9935	2.927697	8
8	6	.8988	.9187	2.160244	8	.9187	.9537	2.256296	8	.9887	.9953	2.806241	7
8	5	.8361	.9176	1.944687	8	.9394	.9534	2.433745	6	.9899	.9953	3.040463	8
8	4	.8909	.9576	2.190883	9	.8909	.9576	2.190883	9	.9899	.9980	2.898269	7
8	3	.8485	.9030	1.854045	5	.9333	.9758	2.553136	5	.9758	.9939	2.686770	5
8	2	.7778	.9333	2.108179	8	.9333	.9778	2.415226	7	.9778	1	3.162275	7
9	9	.8968	1	2.121311	8	.9453	.9659	2.417466	7	.9832	.9906	2.846048	8
9	8	.8786	.9021	2.156283	7	.9436	.9607	2.425815	8	.9899	.9944	2.870958	8
9	7	.8816	.9003	2.095232	7	.9366	.9510	2.378347	7	.9895	.9926	2.984119	8
9	6	.8911	.9253	2.236060	8	.9397	.9694	2.371705	9	.9846	.9939	2.860385	8
9	5	.8871	.9136	2.078696	8	.9406	.9545	2.415229	7	.9860	.9930	2.788859	8
9	4	.8909	.9105	2.303541	7	.9105	.9692	2.306110	9	.9818	.9930	2.962261	7
9	3	.8818	.9227	1.999997	5	.9455	.9818	2.683276	6	.9818	.9955	2.828424	5
9	2	.8000	.9455	2.224853	[illegible]	.9455	.9818	2.553131	8	.9818	1	3.316624	8
10	10	.8774	1	2.108176	7	.9345	.9563	2.344031	8	.9870	.9938	2.927699	8
10	9	.8910	.9158	2.135413	8	.9472	.9580	2.471258	7	.9895	.9918	2.987748	8
10	8	.8805	.9170	2.121317	8	.9369	.9512	2.353388	8	.9894	.9940	2.941739	8
10	7	.8940	.9113	2.226725	7	.9245	.9566	2.281245	9	.9892	.9930	2.886172	7
10	6	.8930	.9188	2.088927	8	.9432	.9537	2.472273	7	.9890	.9955	2.981419	8
10	5	.8565	.9171	2.148343	8	.9171	.9557	2.236067	8	.9730	.9903	2.738606	7
10	4	.8951	.9091	2.366426	6	.9091	.9530	2.415229	9	.9860	.9950	3.089571	8
10	3	.8566	.9056	2.043145	5	.9371	.9545	2.497998	7	.9860	.9965	2.962258	9
10	2	.8182	.9545	2.335492	9	.8182	.9545	2.335492	5	.9848	1	3.464099	6
15	15	.8991	.9017	2.195774	6	.9436	.9549	2.477161	8	.9891	.9926	3.021654	8
15	14	.8888	.9070	2.229665	8	.9496	.9536	2.535291	8	.9882	.9903	2.936018	9
15	13	.8945	.9008	2.289230	7	.9496	.9553	2.572576	8	.9889	.9900	3.012077	9
15	12	.8984	.9138	2.215645	8	.9477	.9554	2.484233	8	.9882	.9900	2.928262	9
15	11	.8977	.9042	2.249427	7	.9485	.9606	2.538947	8	.9888	.9907	2.935798	9
15	10	.8878	.9154	2.261335	7	.9468	.9569	2.499995	8	.9899	.9943	3.061858	8
20	20	.8983	.9048	2.363054	7	.9481	.9524	2.581982	9	.9885	.9904	3.038214	9
20	19	.8970	.9027	2.298753	7	.9497	.9529	2.573541	8	.9894	.9901	3.064835	9
20	18	.8959	.9081	2.276356	8	.9489	.9532	2.558552	8	.9893	.9906	3.019933	9
20	17	.8934	.9090	2.297013	7	.9485	.9522	2.529584	8	.9897	.9903	3.095419	8
20	16	.8972	.9077	2.323786	8	.9499	.9517	2.599197	8	.9898	.9906	3.060678	9
20	15	.8989	.9039	2.261574	8	.9497	.9525	2.538369	8	.9899	.9916	3.107273	8

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
$\theta = 0$ (continued)													
25	25	.8931	.9181	2.357018	8	.9467	.9564	2.611160	9	.9899	.9902	3.111263	9
25	24	.9000	.9027	2.338098	7	.9481	.9505	2.576814	9	.9896	.9902	3.072826	9
25	23	.8980	.9004	2.301713	7	.9487	.9501	2.588380	8	.9899	.9904	3.108690	9
25	22	.8985	.9020	2.271233	8	.9493	.9518	2.558434	8	.9899	.9905	3.095295	9
25	21	.8931	.9002	2.311036	8	.9499	.9513	2.600714	8	.9894	.9903	3.065762	9
25	20	.8913	.9052	2.342606	7	.9465	.9507	2.575179	9	.9894	.9900	3.085770	9
30	30	.8741	.9088	2.335491	8	.9497	.9517	2.587737	9	.9900	.9908	3.151443	9
30	29	.8989	.9014	2.375915	8	.9493	.9538	2.628604	9	.9896	.9900	3.130741	9
30	28	.8989	.9032	2.391197	7	.9486	.9509	2.652709	9	.9900	.9918	3.153222	9
30	27	.8991	.9030	2.373914	8	.9488	.9504	2.610013	9	.9899	.9903	3.123151	9
30	26	.8989	.9005	2.353645	7	.9497	.9509	2.628962	9	.9897	.9903	3.150536	9
30	25	.9000	.9035	2.324660	8	.9487	.9526	2.585344	8	.9896	.9904	3.127595	9
35	35	8979	.9023	2.390457	8	.9488	.9508	2.645749	9	.9899	.9905	3.174896	9
35	34	.8954	.9053	2.355665	8	.9496	.9505	2.617124	9	.9899	.9902	3.165484	9
35	33	.8914	.9004	2.392462	8	.9489	.9506	2.658255	9	.9898	.9904	3.154320	9
35	32	.8991	.9032	2.397930	7	.9497	.9510	2.662829	9	.9897	.9905	3.152208	9
35	31	.8999	.9064	2.417785	7	.9494	.9508	2.669152	9	.9900	.9903	3.181593	9
35	30	.8984	.9054	2.380306	8	.9494	.9508	2.609010	9	.9899	.9902	3.142135	9
40	40	.8977	.9001	2.387041	8	.9489	.9500	2.696795	9	.9896	.9906	3.184468	9
40	39	.8995	.9006	2.406446	8	.9495	.9514	2.664323	9	.9899	.9903	3.182487	9
40	38	.8996	.9030	2.399845	8	.9498	.9507	2.671394	9	.9899	.9901	3.187225	9
40	37	.8950	.9038	2.404320	8	.9498	.9510	2.656782	9	.9898	.9900	3.184932	9
40	36	.8980	.9043	2.421333	8	.9495	.9502	2.657180	9	.9896	.9903	3.183287	9
40	35	.8988	.9010	2.405030	8	.9494	.9504	2.680548	9	.9898	.9903	3.184000	9
45	45	.8998	.9020	2.452760	8	.9491	.9517	2.698643	9	.9898	.9902	3.187669	9
45	44	.8994	.9001	2.439492	7	.9492	.9501	2.692481	9	.9897	.9903	3.200002	9
45	43	.8995	.9004	2.402833	8	.9498	.9504	2.685186	9	.9899	.9902	3.215837	9
45	42	.8990	.9009	2.409033	8	.9499	.9505	2.672514	9	.9896	.9900	3.185898	9
45	41	.8995	.9006	2.421832	8	.9466	.9507	2.660681	9	.9898	.9901	3.210578	9
45	40	.8998	.9079	2.444698	8	.9494	.9531	2.694926	9	.9899	.9900	3.172098	9
50	50	.8948	.9008	2.445997	8	.9488	.9502	2.700650	9	.9899	.9902	3.223286	9
50	49	.8993	.9001	2.451808	8	.9493	.9519	2.716968	9	.9900	.9901	3.227428	9
50	48	.8996	.9013	2.439671	8	.9496	.9501	2.694433	9	.9900	.9902	3.223328	9
50	47	.8991	.9009	2.437650	8	.9497	.9503	2.685562	9	.9898	.9900	3.210537	9
50	46	.8977	.9046	2.426516	8	.9499	.9506	2.693739	9	.9899	.9903	3.243179	9
50	45	.8928	.9010	2.421603	8	.9490	.9501	2.684828	9	.9900	.9902	3.204624	9
100	100	.8998	.9014	2.537297	9	.9491	.9521	2.791446	9	.9899	.9900	3.312942	5
500	500	.8985	.9003	2.691516	5	.9500	.9500	2.963091	5	.9899	.9901	3.474396	5
$\theta = .01$ (see $\theta = 0$ for smaller values of M)													
500	500	.8998	.9010	2.690857	5	.9500	.9500	2.949066	5	.9899	.9901	3.474400	5
$\theta = .05$ (see $\theta = 0$ for smaller values of M)													
100	100	.8997	.9002	2.483005	9	.9496	.9505	2.779514	9	.9899	.9900	3.312943	5
500	500	.8998	.9001	2.547324	5	.9498	.9500	2.842456	9	.9900	.9900	3.407770	5
$\theta = .1$ (see $\theta = 0$ for smaller values of M)													
25	25	.8931	.9181	2.357018	8	.9467	.9564	2.611160	9	.9899	.9902	3.111263	9
25	24	.9000	.9027	2.338098	7	.9481	.9505	2.576814	9	.9896	.9902	3.072826	9
25	23	.8980	.9004	2.301713	7	.9487	.9501	2.588380	8	.9899	.9904	3.108690	9
25	22	.8985	.9020	2.271233	8	.9493	.9518	2.558434	8	.9899	.9905	3.095295	9
25	21	.8931	.9002	2.311036	8	.9499	.9513	2.600714	8	.9894	.9903	3.065762	9
25	20	.8994	.9023	2.267224	8	.9465	.9507	2.575181	8	.9894	.9900	3.085772	9
30	30	.8937	.9088	2.335493	8	.9497	.9517	2.587739	9	.9900	.9908	3.151436	9
30	29	.8988	.9044	2.333530	8	.9493	.9538	2.628599	8	.9896	.9900	3.130745	9
30	28	.8995	.9006	2.350345	8	.9486	.9509	2.652711	8	.9900	.9918	3.153225	9
30	27	.8999	.9025	2.346262	8	.9488	.9504	2.610010	9	.9899	.9903	3.123157	9
30	26	.9000	.9041	2.348687	8	.9497	.9509	2.628958	9	.9897	.9903	3.150541	9
30	25	.8988	.9009	2.302945	8	.9487	.9526	2.585340	8	.9896	.9904	3.127591	9

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
						θ = .1 (continued)							
35	35	.8945	.9026	2.298923	8	.9488	.9508	2.645745	8	.9899	.9905	3.174900	9
35	34	.8989	.9003	2.323115	8	.9496	.9505	2.617124	8	.9899	.9902	3.165483	9
35	33	.8989	.9051	2.347889	8	.9489	.9506	2.658255	8	.9898	.9904	3.154320	9
35	32	.8987	.9015	2.344733	8	.9492	.9501	2.620799	9	.9897	.9905	3.152211	9
35	31	.8996	.9029	2.325303	8	.9486	.9521	2.633547	8	.9900	.9903	3.181589	9
35	30	.9000	.9026	2.316701	8	.9478	.9512	2.603200	8	.9899	.9902	3.142133	9
40	40	.8978	.9002	2.344032	8	.9498	.9517	2.677391	8	.9896	.9906	3.184465	9
40	39	.8994	.9008	2.359977	8	.9491	.9505	2.636461	9	.9899	.9903	3.182486	9
40	38	.8990	.9009	2.349532	8	.9488	.9511	2.623474	9	.9899	.9901	3.187226	9
40	37	.8985	.9033	2.360361	8	.9493	.9500	2.618856	9	.9898	.9900	3.184926	9
40	36	.8990	.9007	2.333967	8	.9490	.9510	2.652736	8	.9896	.9903	3.183289	9
40	35	.8996	.9005	2.360691	8	.9497	.9506	2.643608	9	.9898	.9903	3.184000	9
45	45	.8983	.9001	2.356540	8	.9485	.9500	2.658350	8	.9898	.9902	3.187677	9
45	44	.8996	.9013	2.375515	8	.9496	.9512	2.657539	9	.9897	.9903	3.200002	9
45	43	.8993	.9007	2.370768	8	.9498	.9507	2.632291	9	.9899	.9902	3.215834	9
45	42	.8989	.9020	2.360051	8	.9499	.9505	2.622553	9	.9896	.9900	3.185903	9
45	41	.8996	.9014	2.345000	8	.9497	.9507	2.632239	8	.9898	.9901	3.210579	9
45	40	.8999	.9009	2.334385	8	.9469	.9501	2.658854	8	.9899	.9900	3.172092	9
50	50	.8976	.9001	2.341463	8	.9461	.9510	2.666661	8	.9899	.9902	3.223289	9
50	49	.8979	.9005	2.348361	8	.9499	.9506	2.673242	8	.9900	.9901	3.227426	9
50	48	.8994	.9004	2.363009	8	.9491	.9503	2.644346	9	.9900	.9902	3.223327	9
50	47	.8998	.9029	2.353708	8	.9494	.9506	2.658958	8	.9900	.9901	3.204306	9
50	46	.8997	.9005	2.365886	8	.9498	.9503	2.647974	9	.9898	.9900	3.209180	9
50	45	.8998	.9010	2.347968	8	.9499	.9504	2.663857	8	.9899	.9901	3.192745	9
100	100	.8992	.9001	2.389756	9	.9494	.9502	2.700308	9	.9899	.9901	3.270271	9
500	500	.9000	.9001	2.439167	6	.9500	.9500	2.742827	9	.9900	.9906	3.329183	9
						θ = .25 (see θ = 0 for smaller values of M)							
6	6	.8301	.9286	1.999997	9	.9459	.9870	2.449489	7	.9870	.9989	2.927692	8
6	5	.8810	.9069	2.100524	8	.9459	.9762	2.288686	8	.9870	.9978	2.763848	8
6	4	.8524	.9095	1.844654	8	.9095	.9524	2.108179	6	.9762	.9952	2.581983	7
6	3	.8929	.9524	2.267785	6	.8929	.9524	2.267785	9	.9881	1	2.999994	7
6	2	.8929	.9643	2.108185	5	.8929	.9643	2.108185	9	.9643	1	2.828421	6
7	7	.8939	1	2.160234	8	.9312	.9735	2.366430	8	.9808	.9959	2.788861	8
7	6	.8858	.9266	[illegible]	7	.9266	.9545	2.225389	8	.9808	.9924	2.638993	8
7	5	.8813	.9167	2.070192	7	.9356	.9672	2.366426	7	.9848	.9924	2.898270	8
7	4	.8939	.9394	2.013658	9	.9394	.9667	2.288688	7	.9848	.9970	2.746422	8
7	3	.8333	.9167	1.690306	5	.9167	.9667	2.070194	5	.9667	.9917	2.535461	7
7	2	.8333	.9167	1.792840	9	.9167	.9722	2.267781	6	.9722	1	2.999994	7
8	8	.8738	.9148	2.065590	8	.9148	.9565	2.309394	7	.9740	.9907	2.696796	8
8	7	.8738	.9037	1.901594	7	.9417	.9557	2.351450	6	.9883	.9935	2.977699	8
8	6	.8721	.9038	2.049386	8	.9304	.9537	2.160240	8	.9887	.9953	2.806242	7
8	5	.8827	.9176	1.935250	8	.9394	.9534	2.433743	6	.9899	.9953	3.040461	8
8	4	.8889	.9212	2.070195	9	.9212	.9576	2.165057	8	.9899	.9980	2.898274	6
8	3	.8788	.9394	1.854048	5	.9394	.9758	2.224855	6	.9758	.9939	2.686773	7
8	2	.8667	.9333	1.936489	9	.9333	1	2.415225	6	.9333	1	2.415225	5
9	9	.8476	1	1.620176	6	.9453	.9659	2.417463	9	.9832	.9906	2.846044	8
9	8	.8786	.9021	2.156280	7	.9436	.9607	2.425821	8	.9899	.9944	2.870956	8
9	7	.8843	.9156	2.036692	8	.9366	.9510	2.378349	7	.9895	.9926	2.984122	8
9	6	.8689	.9121	1.906924	9	.9121	.9526	2.236065	6	.9846	.9930	2.860383	7
9	5	.8996	.9271	2.078694	8	.9271	.9570	2.093193	8	.9860	.9930	2.788863	6
9	4	.8741	.9175	1.900291	8	.9399	.9692	2.303541	6	.9818	.9930	2.962259	7
9	3	.8273	.9091	1.924492	5	.9091	.9545	1.999999	7	.9818	.9955	2.828424	6
9	2	.8909	.9455	[illegible]	5	.9455	1	2.553134	7	.9455	1	2.553134	5

$\theta = 0.25$ (continued) $\sqrt{MN/(M+N)}\,W^{+}_{M,N}$

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
10	10	.8121	.9999	1.622872	6	.9345	.9563	2.344033	9	.9870	.9938	2.927695	8
10	9	.8971	.9112	2.082578	8	.9472	.9580	2.471260	7	.9895	.9918	2.987741	8
10	8	.8842	.9071	1.897357	9	.9499	.9648	2.353391	6	.9894	.9940	2.941734	8
10	7	.8837	.9053	2.099489	8	.9417	.9566	2.265029	8	.9892	.9930	2.886173	7
10	6	.8917	.9105	2.065589	8	.9374	.9539	2.367454	7	.9890	.9955	2.981421	8
10	5	.8841	.9281	2.064300	9	.9281	.9697	2.236063	8	.9797	.9903	2.711086	7
10	4	.8701	.9061	1.940214	8	.9371	.9530	2.366431	6	.9850	.9950	2.732512	8
10	3	.8566	.9301	2.043138	8	.9301	.9650	2.133068	8	.9860	.9965	2.962259	6
10	2	.8485	.9091	1.833028	6	.9091	.9545	2.190889	6	.9545	.9999	2.683281	7
15	15	.8810	.9120	1.992044	7	.9489	.9642	2.477167	6	.9896	.9926	2.981417	8
15	14	.8993	.9074	2.107457	8	.9466	.9501	2.410496	7	.9882	.9903	2.936016	8
15	13	.8908	.9018	1.968990	8	.9464	.9575	2.406540	6	.9889	.9900	3.012084	8
15	12	.8915	.9062	2.073320	8	.9481	.9593	2.464747	7	.9871	.9900	2.921499	8
15	11	.8999	.9151	2.050950	8	.9433	.9502	2.327570	7	.9898	.9917	2.935798	7
15	10	.8912	.9041	2.041233	8	.9461	.9580	2.485336	7	.9872	.9909	2.909569	9
20	20	.8980	.9186	2.190884	7	.9421	.9508	2.363055	8	.9881	.9905	2.939387	8
20	19	.8920	.9041	2.106974	7	.9487	.9517	2.401993	7	.9886	.9900	3.028778	8
20	18	.8915	.9062	2.019270	8	.9497	.9577	2.407589	7	.9894	.9903	3.013133	8
20	17	.8977	.9049	2.126196	8	.9474	.9504	2.429543	7	.9900	.9910	3.014924	8
20	16	.8998	.9078	2.103499	8	.9491	.9538	2.371708	8	.9890	.9904	2.993446	8
20	15	.8989	.9121	2.091647	8	.9491	.9577	2.456167	7	.9900	.9911	3.014387	8
25	25	.8952	.9042	2.089776	8	.9498	.9557	2.425355	7	.9898	.9913	3.031689	8
25	24	.8978	.9001	2.138564	8	.9491	.9510	2.442405	8	.9899	.9906	3.025856	8
25	23	.8963	.9039	2.099483	8	.9485	.9509	2.424806	7	.9895	.9904	2.999824	8
25	22	.8993	.9023	2.151128	8	.9496	.9513	2.446070	8	.9898	.9904	3.041746	8
25	21	.8987	.9041	2.053628	8	.9451	.9528	2.374012	7	.9895	.9905	3.018807	8
25	20	.8987	.9042	2.132759	8	.9485	.9534	2.449482	7	.9896	.9907	3.061856	8
30	30	.8852	.9029	2.086996	9	.9462	.9502	2.389753	7	.9896	.9906	3.052801	8
30	29	.8994	.9029	2.107302	8	.9468	.9504	2.422487	7	.9895	.9901	3.010136	8
30	28	.8974	.9001	2.102146	8	.9495	.9520	2.428554	7	.9898	.9905	3.016190	8
30	27	.8973	.9002	2.105926	8	.9482	.9513	2.473088	7	.9896	.9901	3.042513	8
30	26	.8992	.9019	2.143558	8	.9478	.9517	2.398619	8	.9892	.9900	3.003676	8
30	25	.8926	.9014	2.091053	8	.9495	.9513	2.445553	7	.9898	.9903	3.044468	8
35	35	.8984	.9017	2.154931	8	.9470	.9530	2.418963	8	.9894	.9903	3.021655	8
35	34	.8974	.9001	2.110234	8	.9476	.9516	2.445857	7	.9895	.9901	3.039057	8
35	33	.8935	.9015	2.101327	8	.9499	.9512	2.434591	7	.9897	.9902	3.038500	8
35	32	.8982	.9013	2.124724	8	.9481	.9514	2.429496	7	.9900	.9903	3.052166	8
35	31	.8968	.9003	2.137621	8	.9495	.9513	2.441570	8	.9897	.9901	3.016970	8
35	30	.8920	.9015	2.088196	8	.9498	.9511	2.473611	7	.9900	.9904	3.071329	8
40	40	.8979	.9023	2.093161	8	.9500	.9519	2.479113	7	.9894	.9901	3.073744	8
40	39	.8941	.9003	2.135633	8	.9492	.9510	2.440245	8	.9898	.9901	3.059980	8
40	38	.8984	.9003	2.113754	8	.9496	.9529	2.460916	7	.9899	.9902	3.034827	8
40	37	.8985	.9001	2.149213	8	.9493	.9508	2.441905	8	.9899	.9901	3.075087	8
40	36	.8948	.9086	2.122192	8	.9499	.9506	2.484960	7	.9899	.9903	3.055428	8
40	35	.8993	.9018	2.141659	8	.9495	.9507	2.438203	8	.9898	.9902	3.076295	8
45	45	.8986	.9016	2.134163	8	.9497	.9519	2.440097	8	.9896	.9904	3.059410	8
45	44	.8976	.9016	2.116776	8	.9496	.9505	2.461484	7	.9900	.9902	3.079249	8
45	43	.8999	.9014	2.134053	8	.9494	.9504	2.458620	7	.9899	.9902	3.059142	8
45	42	.8992	.9015	2.133396	8	.9496	.9505	2.457687	7	.9899	.9902	3.078839	8
45	41	.8981	.9007	2.138024	8	.9480	.9504	2.421832	8	.9899	.9902	3.048291	8
45	40	.8997	.9015	2.129987	8	.9491	.9500	2.462647	7	.9897	.9904	3.082309	8
50	50	.8980	.9012	2.110997	8	.9490	.9506	2.472254	7	.9899	.9905	3.055048	8
50	49	.8996	.9048	2.140493	8	.9495	.9520	2.486336	7	.9896	.9902	3.065854	8
50	48	.8984	.9004	2.123150	8	.9496	.9504	2.435144	7	.9900	.9903	3.064531	8
50	47	.8996	.9011	2.139685	8	.9487	.9500	2.477804	7	.9899	.9902	3.083601	8
50	46	.8941	.9051	2.123161	8	.9499	.9509	2.455867	7	.9892	.9904	3.066792	8
50	45	.8999	.9019	2.134841	8	.9491	.9505	2.449911	7	.9900	.9902	3.078717	8
100	100	.8996	.9005	2.152650	5	.9500	.9505	2.483001	8	.9899	.9900	3.103202	8
500	500	.8994	.9005	2.190888	6	.9500	.9500	2.522719	8	.9900	.9900	3.136102	8

$\theta = 0$ $\sqrt{MN/(M+N)}W_{M,N}$ 2 - 20

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
2	2	.6667	1	1.999992	7	.6667	1	1.999992	7	.6667	1	1.999992	7
3	3	.7500	.9000	1.732051	6	.9000	1	2.449480	9	.9000	1	2.449480	9
3	2	.8000	1	2.236064	6	.8000	1	2.236064	6	.8000	1	2.236064	6
4	4	.7714	.9714	2.190887	6	.7714	.9714	2.190887	6	.9714	1	2.828422	5
4	3	.8571	.9429	1.984313	8	.9429	.9714	2.645745	6	.9714	1	2.645751	6
4	2	.8667	1	2.449480	9	.8667	1	2.449480	9	.8667	1	2.449480	9
5	5	.7143	.9206	2.070193	9	.9206	.9921	2.581984	7	.9206	.9921	2.581984	7
5	4	.8571	.9206	2.371699	9	.9206	.9841	2.399993	8	.9841	1	2.999999	7
5	3	.8571	.9643	2.190886	5	.8571	.9643	2.190886	5	.9643	1	2.828424	8
5	2	.7143	.9048	1.932183	8	.9048	1	2.645750	6	.9048	1	2.645750	6
6	6	.8918	.9524	2.449480	9	.8918	.9524	2.449480	9	.9740	.9978	2.927695	7
6	5	.8918	.9524	2.288684	5	.8918	.9524	2.288684	5	.9740	.9957	2.763848	9
6	4	.8190	.9048	2.108181	6	.9048	.9524	2.535458	5	.9524	.9905	2.581984	5
6	3	.7857	.9048	2.267783	7	.9048	.9762	2.371701	7	.9762	1	2.999994	5
6	2	.7857	.9286	2.108181	5	.9286	1	2.828422	8	.9286	1	2.828422	8
7	7	.8625	.9470	2.366431	5	.9470	.9615	2.672603	9	.9615	.9918	2.788861	9
7	6	.8551	.9091	2.225394	6	.9324	.9615	2.596289	5	.9883	.9918	3.078826	9
7	5	.8712	.9343	2.366427	6	.9343	.9697	2.474358	6	.9848	.9975	2.927695	5
7	4	.8788	.9333	2.288685	8	.9333	.9515	2.686766	6	.9697	.9939	2.746424	6
7	3	.8333	.9343	2.415220	9	.9333	.9833	2.535457	8	.9833	1	3.162272	7
7	2	.8333	.9444	2.267786	6	.9444	1	2.999999	5	.9444	1	2.999999	5
8	8	.8298	.9130	2.309397	6	.9479	.9814	2.696795	5	.9869	.9975	3.098383	9
8	7	.8634	.9114	2.351452	6	.9441	.9557	2.561731	6	.9869	.9953	2.958038	5
8	6	.8378	.9074	2.156299	7	.9394	.9554	2.650336	6	.9840	.9907	2.806243	6
8	5	.8788	.9068	2.433745	8	.9068	.9580	2.497996	7	.9798	.9907	3.040462	6
8	4	.7838	.9152	2.190889	5	.9152	.9515	2.449480	9	.9798	.9960	2.898269	7
8	3	.8667	.9515	2.553134	5	.8667	.9515	2.553134	5	.9879	.9939	3.316616	8
8	2	.8667	.9556	2.415227	7	.8667	.9556	2.415227	7	.9556	1	3.162272	6
9	9	.8906	.9517	2.417466	7	.9317	.9664	2.631173	6	.9812	.9905	2.999994	5
9	8	.8872	.9114	2.425820	7	.9441	.9552	2.671571	6	.9888	.9923	3.149589	5
9	7	.8733	.9021	2.378350	8	.9399	.9659	2.618610	7	.9851	.9925	3.057878	6
9	6	.8799	.9389	2.371699	9	.9389	.9580	2.581984	8	.9860	.9940	2.958034	7
9	5	.8811	.9091	2.415220	9	.9301	.9720	2.621583	5	.9860	.9940	3.174900	7
9	4	.8224	.9385	2.306117	6	.9385	.9636	2.596293	5	.9860	.9972	3.040460	9
9	3	.8909	.9636	2.683280	6	.8909	.9636	2.683280	6	.9636	.9909	2.828422	6
9	2	.8909	.9636	2.553138	9	.8909	.9636	2.553138	9	.9636	1	3.316618	7
10	10	.8690	.9126	2.344035	8	.9476	.9554	2.683277	7	.9877	.9935	3.146266	6
10	9	.8944	.9131	2.471263	8	.9426	.9562	2.595912	7	.9886	.9930	3.121472	6
10	8	.8738	.9025	2.353389	9	.9217	.9501	2.535458	8	.9880	.9930	3.027148	7
10	7	.8496	.9131	2.281241	5	.9445	.9574	2.671569	8	.9860	.9901	3.121778	7
10	6	.8865	.9075	2.472179	5	.9075	.9580	2.480694	5	.9780	.9910	2.981418	8
10	5	.8342	.9114	2.236064	6	.9461	.9807	2.738608	5	.9807	.9900	2.927695	9
10	4	.8192	.9061	2.415225	6	.9061	.9540	2.432075	6	.9720	.9900	3.089570	5
10	3	.8741	.9091	2.497991	8	.9091	.9720	2.896917	7	.9720	.9930	2.962258	6
10	2	.6515	.9091	2.335496	6	.9091	.9697	2.683280	5	.9697	1	3.464100	8
15	15	.8872	.9099	2.477166	5	.9399	.9564	2.738604	9	.9899	.9908	3.286331	9
15	14	.8993	.9072	2.535290	8	.9496	.9531	2.782793	9	.9893	.9909	3.195060	5
15	13	.8992	.9106	2.572578	7	.9432	.9505	2.654929	5	.9880	.9903	3.115546	9
15	12	.8954	.9107	2.484228	7	.9467	.9523	2.762508	9	.9896	.9910	3.240497	5
15	11	.8972	.9213	2.538949	8	.9434	.9500	2.688572	5	.9894	.9911	3.177120	9
15	10	.8936	.9139	2.499996	8	.9342	.9545	2.672609	5	.9886	.9913	3.105164	9
20	20	.8964	.9049	2.581982	7	.9431	.9505	2.760260	5	.9895	.9915	3.227481	5
20	19	.8995	.9059	2.573546	7	.9480	.9510	2.766439	5	.9893	.9906	3.254874	5
20	18	.8979	.9065	2.558548	8	.9449	.9507	2.778952	5	.9897	.9907	3.257878	5
20	17	.8971	.9044	2.529583	8	.9476	.9505	2.773016	9	.9888	.9909	3.186979	5
20	16	.8999	.9035	2.591198	7	.9473	.9506	2.774995	5	.9892	.9908	3.295764	5
20	15	.8994	.9051	2.538373	8	.9374	.9585	2.788865	9	.9900	.9918	3.237672	5

θ = 0 (continued)

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
25	25	.8936	.9129	2.611160	8	.9467	.9501	2.849009	5	.9889	.9903	3.311330	5
25	24	.8963	.9011	2.576818	8	.9494	.9517	2.805807	5	.9898	.9904	3.299827	5
25	23	.8976	.9004	2.588380	8	.9493	.9521	2.835793	5	.9900	.9908	3.268822	5
25	22	.8987	.9036	2.558433	8	.9489	.9531	2.813671	9	.9895	.9901	3.323179	5
25	21	.8998	.9028	2.600712	8	.9485	.9522	2.852145	5	.9898	.9906	3.276172	5
25	20	.8931	.9015	2.575178	8	.9476	.9507	2.794653	5	.9895	.9906	3.287310	5
30	30	.8995	.9035	2.587738	8	.9496	.9517	2.841766	5	.9899	.9905	3.358451	5
30	29	.8987	.9077	2.628599	8	.9489	.9507	2.849403	5	.9897	.9905	3.314543	5
30	28	.8973	.9020	2.652715	7	.9485	.9506	2.889139	5	.9897	.9901	3.355934	5
30	27	.8978	.9010	2.610014	8	.9488	.9504	2.831194	5	.9898	.9904	3.308399	5
30	26	.8996	.9020	2.628961	8	.9470	.9505	2.844097	5	.9894	.9903	3.299850	5
30	25	.8975	.9053	2.585348	8	.9449	.9506	2.842818	5	.9897	.9906	3.351719	5
35	35	.8977	.9018	2.645746	8	.9491	.9510	2.879140	5	.9893	.9901	3.380897	5
35	34	.8993	.9011	2.617128	8	.9492	.9502	2.887846	5	.9900	.9904	3.357211	5
35	33	.8979	.9014	2.658252	8	.9496	.9515	2.894682	5	.9900	.9904	3.351746	5
35	32	.8996	.9021	2.662826	8	.9486	.9517	2.882021	5	.9898	.9903	3.360711	5
35	31	.8989	.9017	2.669152	8	.9498	.9514	2.915909	5	.9896	.9900	3.363868	5
35	30	.8990	.9017	2.609014	8	.9500	.9515	2.861443	5	.9899	.9905	3.352086	5
40	40	.8980	.9002	2.696796	8	.9470	.9506	2.921908	5	.9895	.9909	3.380612	5
40	39	.8993	.9029	2.664318	8	.9487	.9503	2.894423	5	.9899	.9901	3.376525	5
40	38	.8999	.9016	2.671396	8	.9496	.9524	2.910080	5	.9899	.9901	3.393253	5
40	37	.8999	.9021	2.656786	8	.9484	.9523	2.885186	5	.9898	.9901	3.358492	5
40	36	.8992	.9007	2.657180	8	.9493	.9527	2.926911	5	.9898	.9902	3.377388	5
40	35	.8991	.9009	2.680551	8	.9496	.9505	2.903017	5	.9899	.9902	3.374005	5
45	45	.8984	.9037	2.698643	8	.9491	.9508	2.943310	5	.9897	.9902	3.383358	5
45	44	.8987	.9005	2.692477	8	.9493	.9501	2.930842	5	.9899	.9904	3.394387	5
45	43	.8998	.9011	2.685182	8	.9473	.9501	2.899806	5	.9899	.9900	3.409947	5
45	42	.8999	.9011	2.672515	8	.9497	.9504	2.915055	5	.9900	.9902	3.393191	5
45	41	.8935	.9015	2.660681	8	.9493	.9500	2.895261	5	.9897	.9900	3.390005	5
45	40	.8989	.9064	2.694930	8	.9495	.9501	2.916282	5	.9899	.9902	3.375119	5
50	50	.8978	.9007	2.700655	8	.9498	.9547	2.948834	5	.9899	.9903	3.429972	5
50	49	.8988	.9041	2.716969	8	.9496	.9503	2.935704	5	.9900	.9902	3.418823	5
50	48	.8994	.9004	2.694438	8	.9499	.9505	2.934915	5	.9898	.9901	3.407365	5
50	47	.8996	.9007	2.685570	8	.9498	.9505	2.944334	5	.9899	.9902	3.412756	5
50	46	.8991	.9001	2.687647	8	.9496	.9503	2.930365	5	.9898	.9900	3.428852	5
50	45	.8982	.9003	2.684827	8	.9497	.9503	2.929361	5	.9898	.9901	3.408348	5
100	100	.8936	.9044	2.791449	8	.9499	.9502	3.024948	5	.9899	.9900	3.502189	5
500	500	.8999	.9003	2.961849	5	.9500	.9500	3.186422	5	.9899	.9900	3.669355	5

θ = .01 (see θ = 0 for smaller values of M)

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
500	500	.8999	.9000	2.944097	5	.9500	.9500	3.186421	5	.9899	.9900	3.669354	5

θ = .05 (see θ = 0 for smaller values of M)

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
100	100	.8996	.9012	2.779509	8	.9497	.9501	3.008865	5	.9899	.9900	3.502189	5
500	500	.9000	.9003	2.842462	5	.9500	.9500	3.099087	5	.9900	.9900	3.613597	5

θ = .1 (see θ = 0 for smaller values of M)

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
35	35	.8977	.9018	2.645746	8	.9491	.9510	2.879140	5	.9893	.9901	3.380897	5
35	34	.8993	.9011	2.617128	8	.9492	.9502	2.887846	5	.9900	.9904	3.357211	5
35	33	.8979	.9014	2.658252	8	.9496	.9515	2.894682	5	.9900	.9904	3.351746	5
35	32	.8985	.9003	2.620807	8	.9486	.9517	2.882018	5	.9898	.9903	3.360707	5
35	31	.8974	.9042	2.633544	8	.9498	.9514	2.915910	5	.9896	.9900	3.363869	5
35	30	.8957	.9024	2.603207	8	.9500	.9515	2.861439	5	.9899	.9905	3.352087	5
40	40	.8996	.9035	2.677397	8	.9470	.9506	2.921906	5	.9895	.9909	3.380615	5
40	39	.8983	.9011	2.636463	8	.9487	.9503	2.894420	5	.9899	.9901	3.376522	5
40	38	.8978	.9024	2.623478	8	.9496	.9524	2.910078	5	.9899	.9901	3.393251	5
40	37	.8988	.9002	2.618862	8	.9500	.9509	2.880481	5	.9898	.9901	3.358491	5
40	36	.8981	.9021	2.652736	8	.9493	.9519	2.897329	5	.9898	.9902	3.377389	5
40	35	.8996	.9014	2.643610	8	.9486	.9516	2.893184	5	.9899	.9902	3.374004	5
45	45	.8971	.9002	2.658346	8	.9493	.9525	2.908373	5	.9897	.9902	3.383359	5
45	44	.8993	.9024	2.657541	8	.9492	.9503	2.900927	5	.9899	.9904	3.394385	5
45	43	.8997	.9015	2.632293	8	.9491	.9503	2.871079	5	.9899	.9900	3.409946	5
45	42	.8999	.9012	2.622553	8	.9499	.9506	2.896045	5	.9900	.9902	3.393189	5
45	41	.8995	.9016	2.632237	8	.9487	.9507	2.894458	5	.9897	.9900	3.390004	5
45	40	.8940	.9003	2.658851	8	.9496	.9514	2.900130	5	.9899	.9902	3.375118	5
50	50	.8924	.9021	2.666666	8	.9496	.9515	2.896827	5	.9899	.9903	3.429970	5
50	49	.8999	.9013	2.673243	8	.9497	.9501	2.914944	5	.9900	.9902	3.418822	5
50	48	.8984	.9008	2.644348	8	.9492	.9503	2.905985	5	.9898	.9901	3.407367	5
50	47	.8989	.9014	2.658960	8	.9495	.9500	2.934078	5	.9899	.9902	3.412753	5
50	46	.8998	.9008	2.647982	8	.9497	.9509	2.918853	5	.9898	.9900	3.428854	5
50	45	.9000	.9009	2.663861	8	.9484	.9503	2.897344	5	.9898	.9901	3.408347	5
100	100	.8996	.9005	2.700306	8	.9500	.9503	2.971183	5	.9900	.9901	3.4[illegible]2694	5
500	500	.9000	.9001	2.741619	5	.9500	.9500	3.013864	5	.9900	.9900	3.546824	5

θ = .25 (see θ = 0 for smaller values of M)

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
8	8	.8298	.9130	2.309397	6	.9479	.9814	2.696795	5	.9869	.9975	3.098387	9
8	7	.8834	.9114	2.351452	6	.9441	.9557	2.561731	6	.9859	.9953	2.956038	5
8	6	.8608	.9074	2.160239	8	.9394	.9554	2.650336	6	.9840	.9907	2.806243	6
8	5	.8788	.9068	2.433745	8	.9068	.9580	2.497996	7	.9798	.9907	3.040462	6
8	4	.8788	.9152	2.165064	5	.9152	.9515	2.439480	9	.9798	.9960	2.893269	7
8	3	.8788	.9515	2.224858	6	.8788	.9515	2.224858	6	.9879	.9939	3.316616	9
8	2	.8667	1	2.415227	7	.8667	1	2.415227	7	.8667	1	2.415227	7
9	9	.8906	.9317	2.417466	7	.9317	.9664	2.631173	6	.9812	.9905	2.999994	5
9	8	.8873	.9214	2.425820	7	.9441	.9552	2.671571	6	.9888	.9923	3.145589	5
9	7	.8733	.9021	2.378350	8	.9399	.9659	2.618610	7	.9851	.9925	3.057878	6
9	6	.8242	.9053	2.236064	9	.9389	.9580	2.581984	8	.9860	.9940	2.953034	7
9	5	.8541	.9141	2.093198	5	.9441	.9720	2.577578	9	.9860	.9940	3.174900	7
9	4	.8797	.9385	2.303545	6	.9385	.9636	2.596293	5	.9860	.9972	3.040460	9
9	3	.8182	.9091	1.999996	8	.9091	.9636	2.366426	7	.9636	.9909	2.825422	6
9	2	.8909	1	2.553138	9	.8909	1	2.553138	9	.8909	1	2.553138	9

θ = 0.25 (continued) $\sqrt{MN/(M+N)}W_{M,N}$ 10 - 500

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
10	10	.8690	.9126	2.344035	8	.9476	.9554	2.683277	7	.9877	.9935	3.146266	6
10	9	.8944	.9161	2.471263	8	.9426	.9562	2.595912	7	.9886	.9930	3.121477	6
10	8	.8998	.9296	2.353389	9	.9296	.9501	2.439517	8	.9880	.9930	3.027148	7
10	7	.8835	.9131	2.265026	5	.9445	.9574	2.671569	8	.9860	.9901	3.121778	7
10	6	.8749	.9078	2.367453	5	.9336	.9580	2.472279	5	.9780	.9910	2.981418	8
10	5	.8561	.9394	2.236064	6	.9394	.9594	2.561732	5	.9807	.9900	2.927695	9
10	4	.8741	.9061	2.366431	7	.9061	.9700	2.432075	6	.9700	.9900	2.732519	6
10	3	.8601	.9301	2.133068	9	.9301	.9720	2.497991	8	.9720	.9930	2.962258	6
10	2	.8182	.9091	2.190889	6	.9091	1	2.683280	5	.9091	1	2.683280	5
15	15	.8979	.9284	2.477166	5	.9472	.9611	2.652067	9	.9899	.9908	3.286329	9
15	14	.8932	.9003	2.410500	8	.9489	.9559	2.686729	9	.9893	.9909	3.195057	9
15	13	.8929	.9150	2.406542	8	.9467	.9542	2.654925	9	.9880	.9903	3.115552	9
15	12	.8962	.9185	2.464749	7	.9455	.9535	2.598076	5	.9896	.9910	3.240499	8
15	11	.8866	.9003	2.327574	8	.9497	.9565	2.688570	8	.9895	.9921	3.123849	9
15	10	.8923	.9160	2.485337	7	.9378	.9502	2.545871	5	.9886	.9913	3.105170	8
20	20	.8842	.9017	2.363055	8	.9428	.9520	2.651972	9	.9877	.9907	3.186960	9
20	19	.8975	.9034	2.401996	8	.9483	.9508	2.718804	9	.9892	.9906	3.171417	5
20	18	.8994	.9155	2.407591	8	.9467	.9511	2.648053	9	.9892	.9900	3.232690	9
20	17	.8948	.9008	2.429541	8	.9437	.9500	2.670118	9	.9889	.9902	3.180658	9
20	16	.8981	.9077	2.371702	8	.9491	.9574	2.662556	9	.9891	.9904	3.174900	9
20	15	.8981	.9155	2.456166	8	.9458	.9506	2.644655	9	.9899	.9909	3.231542	9
25	25	.8996	.9114	2.425349	8	.9472	.9537	2.686858	9	.9896	.9913	3.204932	9
25	24	.8981	.9021	2.442400	8	.9494	.9549	2.739560	9	.9892	.9900	3.253304	9
25	23	.8970	.9018	2.424800	8	.9473	.9502	2.686053	9	.9895	.9903	3.213433	9
25	22	.8992	.9026	2.446069	8	.9450	.9500	2.669670	9	.9894	.9908	3.211944	9
25	21	.8901	.9056	2.374012	8	.9490	.9551	2.672463	9	.9898	.9907	3.244117	9
25	20	.8971	.9068	2.449483	8	.9469	.9503	2.736085	9	.9896	.9905	3.203240	5
30	30	.8923	.9003	2.389747	8	.9427	.9532	2.683279	9	.9891	.9908	3.279558	9
30	29	.8936	.9007	2.422489	8	.9483	.9500	2.738250	9	.9900	.9905	3.262783	9
30	28	.8990	.9041	2.428550	8	.9485	.9507	2.689643	9	.9894	.9900	3.220249	9
30	27	.8963	.9026	2.473087	8	.9485	.9507	2.720708	9	.9896	.9902	3.293590	9
30	26	.8956	.9034	2.398621	8	.9492	.9512	2.724689	9	.9896	.9904	3.237465	9
30	25	.8990	.9027	2.445555	8	.9493	.9508	2.763533	9	.9896	.9902	3.226615	5
35	35	.8941	.9061	2.418958	8	.9481	.9560	2.734700	9	.9894	.9910	3.281647	9
35	34	.8953	.9032	2.445854	8	.9489	.9511	2.730650	9	.9895	.9901	3.261389	9
35	33	.8998	.9024	2.434595	8	.9492	.9508	2.716451	9	.9898	.9902	3.248118	9
35	32	.8963	.9027	2.429492	8	.9480	.9523	2.743250	9	.9898	.9903	3.296289	9
35	31	.8991	.9026	2.441569	8	.9489	.9506	2.712996	9	.9896	.9901	3.242127	9
35	30	.8996	.9022	2.473611	8	.9483	.9502	2.709942	9	.9900	.9903	3.301233	9
40	40	.9000	.9038	2.479115	8	.9479	.9517	2.717283	9	.9896	.9903	3.281650	9
40	39	.8984	.9019	2.440243	8	.9494	.9520	2.733965	9	.9896	.9900	3.273561	9
40	38	.8992	.9058	2.460918	8	.9489	.9527	2.726968	9	.9894	.9902	3.245753	9
40	37	.8986	.9015	2.441905	8	.9496	.9513	2.724457	9	.9900	.9903	3.296534	9
40	36	.8998	.9012	2.484957	8	.9491	.9506	2.732196	9	.9897	.9901	3.252646	9
40	35	.8991	.9014	2.438204	8	.9493	.9507	2.727643	9	.9897	.9901	3.273266	9
45	45	.8994	.9038	2.440091	8	.9496	.9502	2.740637	9	.9893	.9901	3.255910	9
45	44	.8993	.9010	2.461486	8	.9494	.9510	2.729247	9	.9899	.9902	3.297359	9
45	43	.8989	.9008	2.458619	8	.9499	.9506	2.764376	9	.9898	.9904	3.281685	9
45	42	.8992	.9009	2.457682	8	.9494	.9520	2.730768	9	.9898	.9902	3.286086	9
45	41	.8961	.9008	2.421829	8	.9472	.9508	2.727269	9	.9900	.9903	3.262100	9
45	40	.8983	.9001	2.462645	8	.9490	.9506	2.718667	9	.9895	.9900	3.297916	9
50	50	.8981	.9012	2.472254	8	.9481	.9523	2.735755	9	.9900	.9904	3.296338	9
50	49	.8990	.9041	2.486332	8	.9500	.9514	2.741131	9	.9900	.9901	3.315270	9
50	48	.8993	.9008	2.435141	8	.9496	.9508	2.726158	9	.9900	.9902	3.282599	9
50	47	.8974	.9000	2.477804	8	.9492	.9501	2.739743	9	.9897	.9902	3.304186	9
50	46	.8998	.9018	2.455865	8	.9498	.9509	2.744790	9	.9898	.9901	3.268825	9
50	45	.8982	.9010	2.449909	8	.9500	.9509	2.737624	9	.9899	.9903	3.284911	9
100	100	.8999	.9011	2.483005	9	.9486	.9506	2.757936	9	.9898	.9900	3.313039	9
500	500	.8999	.9000	2.522724	5	.9500	.9501	2.802861	9	.9900	.9900	3.362302	5

See $W^{+}_{M,N}$ for $\theta = 0$.

θ = .01 (for smaller values of M see θ = 0)

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
100	100	.8996	.9001	2.519758	9	.9490	.9503	2.789055	9	.9899	.9900	3.311330	5
500	500	.9000	.9000	2.661939	5	.9495	.9501	2.926526	5	.9900	.9900	3.463385	5

θ = .05 (for smaller values of M see θ = 0)

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
10	10	.8503	.9016	1.951796	9	.9406	.9672	2.344035	8	.9802	.9912	2.738606	6
10	9	.8869	.9107	2.132999	9	.9365	.9517	2.205804	9	.9865	.9912	2.809325	7
10	8	.8776	.9071	2.028367	5	.9351	.9529	2.347866	9	.9849	.9905	2.810920	7
10	7	.8837	.9107	2.099487	5	.9488	.9642	2.265026	5	.9838	.9913	2.671569	8
10	6	.8917	.9105	2.065590	6	.9374	.9580	2.367453	5	.9843	.9905	2.732514	9
10	5	.8641	.9281	2.064304	6	.9281	.9697	2.236064	6	.9797	.9930	2.711086	5
10	4	.8701	.9061	1.940216	8	.9371	.9530	2.366431	7	.9850	.9950	2.732519	6
10	3	.8566	.9301	2.043142	5	.9301	.9650	2.133068	9	.9860	1	2.962258	6
10	2	.8485	.9091	1.833026	7	.9091	.9545	2.190889	6	.9545	1	2.683280	5
15	15	.8771	.9019	2.158327	6	.9463	.9595	2.477168	8	.9884	.9913	2.981418	8
15	14	.8953	.9050	2.132762	8	.9458	.9504	2.419674	8	.9891	.9918	2.936014	8
15	13	.8969	.9017	2.244526	7	.9411	.9520	2.406536	8	.9895	.9906	3.012080	8
15	12	.8951	.9147	2.173700	8	.9413	.9515	2.464746	8	.9871	.9905	2.921498	8
15	11	.8967	.9054	2.132315	7	.9484	.9580	2.391650	8	.9898	.9917	2.935793	8
15	10	.8431	.9041	2.041238	8	.9461	.9580	2.485336	7	.9872	.9916	2.909568	9
20	20	.8953	.9024	2.238921	7	.9493	.9535	2.542560	8	.9895	.9914	3.038211	9
20	19	.8975	.9015	2.265293	7	.9483	.9516	2.491826	8	.9894	.9901	3.048044	8
20	18	.8967	.9000	2.260279	7	.9450	.9511	2.532288	8	.9897	.9909	3.016519	9
20	17	.8947	.9088	2.216761	7	.9433	.9504	2.467200	8	.9900	.9910	3.014925	8
20	16	.8983	.9017	2.250078	7	.9416	.9508	2.448275	8	.9890	.9904	2.993440	8
20	15	.8862	.9030	2.200980	7	.9465	.9501	2.484281	8	.9900	.9911	3.014382	8
25	25	.8946	.9078	2.314548	7	.9436	.9507	2.576730	8	.9893	.9903	3.086059	9
25	24	.8957	.9066	2.312022	7	.9477	.9531	2.561975	8	.9899	.9906	3.072562	9
25	23	.8985	.9035	2.282523	7	.9491	.9520	2.563548	8	.9899	.9907	3.101819	9
25	22	.8994	.9022	2.251872	8	.9470	.9528	2.532099	8	.9893	.9905	3.085049	8
25	21	.8972	.9072	2.311037	7	.9482	.9504	2.586718	8	.9895	.9902	3.061097	9
25	20	.8989	.9031	2.261332	8	.9486	.9505	2.533952	8	.9898	.9905	3.063967	8
30	30	.8910	.9189	2.355492	8	.9433	.9548	2.581987	9	.9898	.9904	3.105290	9
30	29	.8979	.9091	2.298007	8	.9445	.9501	2.540984	8	.9899	.9903	3.088230	8
30	28	.8882	.9000	2.259844	8	.9497	.9511	2.573535	8	.9900	.9904	3.126374	9
30	27	.8964	.9043	2.275928	8	.9457	.9500	2.547256	8	.9897	.9903	3.075512	9
30	26	.8977	.9019	2.318970	7	.9485	.9506	2.561227	9	.9898	.9903	3.073543	9
30	25	.8972	.9010	2.265053	8	.9500	.9518	2.547568	8	.9897	.9903	3.089812	9
35	35	.8943	.9048	2.320476	6	.9493	.9523	2.621588	8	.9898	.9904	3.154283	9
35	34	.8994	.9020	2.334302	8	.9486	.9500	2.583534	9	.9898	.9902	3.121389	9
35	33	.8999	.9015	2.305586	8	.9491	.9511	2.639673	8	.9896	.9903	3.127321	9
35	32	.8999	.9016	2.339790	8	.9497	.9517	2.594919	9	.9900	.9904	3.098438	9
35	31	.8964	.9004	2.319412	7	.9491	.9508	2.581090	8	.9900	.9905	3.156600	9
35	30	.8993	.9010	2.321045	8	.9467	.9504	2.593092	8	.9887	.9909	3.110074	9
40	40	.8977	.9001	2.344035	8	.9498	.9514	2.653296	8	.9893	.9903	3.150901	9
40	39	.8998	.9005	2.360296	8	.9500	.9515	2.636463	9	.9898	.9901	3.147721	9
40	38	.8992	.9003	2.361987	8	.9492	.9512	2.623473	9	.9897	.9900	3.162393	9
40	37	.8977	.9005	2.368448	8	.9494	.9506	2.635704	9	.9899	.9902	3.131933	9
40	36	.8997	.9029	2.344043	8	.9499	.9532	2.634218	8	.9898	.9904	3.181455	9
40	35	.8981	.9002	2.362273	8	.9494	.9502	2.603862	9	.9898	.9903	3.126280	9

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
						θ = .05 (continued)							
45	45	.8985	.9003	2.356540	8	.9489	.9505	2.658350	8	.9898	.9901	3.165403	9
45	44	.8990	.9006	2.375511	8	.9490	.9504	2.657534	9	.9900	.9902	3.169887	9
45	43	.8999	.9009	2.377790	8	.9499	.9514	2.648713	9	.9899	.9900	3.198830	9
45	42	.9000	.9051	2.381935	8	.9497	.9502	2.663688	9	.9898	.9901	3.170070	9
45	41	.8995	.9006	2.363058	8	.9480	.9513	2.634932	9	.9899	.9901	3.183469	9
45	40	.8996	.9013	2.377835	8	.9497	.9507	2.661448	9	.9900	.9906	3.170569	9
50	50	.8979	.9014	2.395955	8	.9494	.9512	2.666658	9	.9896	.9901	3.191678	9
50	49	.8989	.9002	2.363861	8	.9498	.9510	2.653292	9	.9896	.9901	3.174482	9
50	48	.8993	.9033	2.392378	8	.9498	.9510	2.644348	9	.9898	.9902	3.197502	9
50	47	.8994	.9004	2.374360	8	.9482	.9504	2.658960	9	.9899	.9902	3.177045	9
50	46	.8985	.9007	2.375526	8	.9469	.9501	2.635617	9	.9898	.9900	3.160956	9
50	45	.8990	.9000	2.372050	8	.9498	.9506	2.640609	9	.9899	.9900	3.179711	9
100	100	.8999	.9010	2.476525	9	.9497	.9500	2.736857	9	.9900	.9901	3.280524	9
500	500	.9000	.9000	2.546704	5	.9500	.9501	2.838951	9	.9900	.9900	3.403127	5
						θ = .1 (see θ = 0 for smaller values of M)							
5	5	.8571	.9167	1.897363	5	.9167	.9762	2.070193	9	.9762	1	2.581984	7
5	4	.8810	.9603	1.897366	5	.8810	.9603	1.897366	5	.9603	1	2.399993	8
5	3	.8214	.9286	1.697055	7	.9286	1	2.190886	5	.9286	1	2.190886	5
5	2	.8571	1	1.932183	8	.8571	1	1.932183	8	.8571	1	1.932183	8
6	6	.8896	.9437	1.999996	5	.9437	.9697	2.309397	9	.9697	.9924	2.449480	9
6	5	.8463	.9113	1.854047	6	.9113	.9545	2.100528	5	.9870	1	2.763848	9
6	4	.8524	.9286	1.844657	7	.9286	.9762	2.108181	6	.9762	1	2.581984	5
6	3	.8810	.9524	1.897361	9	.8810	.9524	1.897361	9	.9524	1	2.371701	7
6	2	.8929	1	2.108181	5	.8929	1	2.108181	5	.8929	1	2.108181	5
7	7	.8584	.9079	1.954012	6	.9079	.9569	2.160243	6	.9895	.9977	2.788861	9
7	6	.8613	.9021	1.935261	7	.9371	.9662	2.225394	6	.9837	.9959	2.638991	5
7	5	.8535	.9040	1.756613	9	.9419	.9735	2.276409	7	.9735	.9924	2.474358	6
7	4	.8939	.9545	2.013658	9	.8939	.9545	2.013658	9	.9848	1	2.746424	6
7	3	.8333	.9167	1.690308	6	.9167	.9667	2.070193	5	.9667	1	2.535457	8
7	2	.8333	.9167	1.792838	8	.9167	1	2.267786	6	.9167	1	2.267786	6
8	8	.8877	.9406	2.065590	7	.9406	.9623	2.309397	6	.9841	.9928	2.696795	5
8	7	.8834	.9145	1.901588	8	.9409	.9596	2.324172	7	.9879	.9944	2.835575	5
8	6	.8721	.9038	2.049383	8	.9371	.9620	2.160239	8	.9787	.9907	2.650336	6
8	5	.8982	.9371	1.935261	5	.9371	.9604	2.253461	8	.9837	.9953	2.638987	7
8	4	.8889	.9212	2.070193	5	.9212	.9697	2.165058	5	.9899	1	2.898269	7
8	3	.8788	.9394	1.854042	7	.9394	.9758	2.224858	6	.9758	1	2.686768	5
8	2	.8667	.9333	1.936482	9	.9333	1	2.415227	7	.9333	1	2.415227	7
9	9	.8691	.9218	1.999996	8	.9427	.9514	2.357021	7	.9855	.9944	2.846046	6
9	8	.8893	.9134	2.156285	8	.9492	.9650	2.265026	8	.9843	.9912	2.691467	6
9	7	.8920	.9240	2.036694	9	.9458	.9607	2.378350	8	.9858	.9926	2.732519	7
9	6	.8689	.9121	1.906921	5	.9121	.9580	2.236064	9	.9860	.9944	2.797154	7
9	5	.8996	.9271	2.078693	5	.9271	.9570	2.093198	5	.9895	.9970	2.788864	8
9	4	.8741	.9175	1.900286	7	.9399	.9790	2.303545	6	.9790	.9930	2.596293	5
9	3	.8273	.9091	1.924499	8	.9091	.9545	1.999996	8	.9818	1	2.828422	6
9	2	.8909	.9455	2.068277	5	.9455	1	2.553138	9	.9455	1	2.553138	9
10	10	.8503	.9016	1.951796	9	.9406	.9672	2.344035	8	.9802	.9912	2.738606	6
10	9	.8869	.9107	2.132999	9	.9365	.9517	2.205804	9	.9865	.9912	2.809325	7
10	8	.8776	.9071	2.028367	5	.9351	.9529	2.347866	9	.9849	.9905	2.810920	7
10	7	.8837	.9107	2.099487	5	.9488	.9642	2.265026	5	.9838	.9913	2.671569	8
10	6	.8917	.9105	2.065590	6	.9374	.9580	2.367453	5	.9843	.9905	2.732514	9
10	5	.8841	.9281	2.064304	6	.9281	.9697	2.236064	6	.9797	.9930	2.711088	5
10	4	.8701	.9061	1.940216	8	.9371	.9530	2.366431	7	.9850	.9950	2.732519	6
10	3	.8566	.9301	2.043142	5	.9301	.9650	2.133068	9	.9860	1	2.962258	6
10	2	.8485	.9091	1.833026	7	.9091	.9545	2.190889	6	.9545	1	2.683280	5

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
15	15	.8957	.9064	2.158327	6	.9464	.9559	2.449485	8	.9895	.9931	2.981417	8
15	14	.8968	.9047	2.107457	8	.9499	.9548	2.419674	7	.9882	.9903	2.861979	8
15	13	.8946	.9037	2.090051	7	.9484	.9551	2.306549	8	.9887	.9908	2.880877	7
15	12	.8935	.9031	2.078458	7	.9476	.9619	2.464746	7	.9887	.9912	2.857883	9
15	11	.8900	.9078	2.130648	7	.9460	.9528	2.342727	8	.9894	.9915	2.911020	8
15	10	.8766	.9122	2.041241	8	.9383	.9504	2.294150	7	.9886	.9924	2.891756	7
20	20	.8983	.9044	2.216363	7	.9432	.9523	2.478744	8	.9877	.9901	2.939383	8
20	19	.8961	.9021	2.189138	7	.9476	.9504	2.471597	8	.9898	.9906	3.048050	8
20	18	.8972	.9001	2.251189	8	.9484	.9522	2.493685	8	.9896	.9906	3.016349	8
20	17	.8895	.9019	2.202753	8	.9468	.9539	2.467197	8	.9889	.9905	2.945480	8
20	16	.8994	.9040	2.216962	7	.9449	.9533	2.399995	8	.9891	.9902	2.986033	8
20	15	.8557	.9021	2.091649	8	.9438	.9523	2.456161	7	.9897	.9908	2.981165	8
25	25	.8988	.9027	2.264553	7	.9491	.9502	2.545580	8	.9898	.9906	3.072548	9
25	24	.8955	.9012	2.256420	8	.9499	.9517	2.525612	8	.9895	.9907	3.029603	9
25	23	.8966	.9010	2.248475	7	.9486	.9507	2.504569	8	.9896	.9910	2.999823	8
25	22	.8948	.9081	2.218961	7	.9483	.9503	2.475999	8	.9895	.9901	3.041746	8
25	21	.8967	.9008	2.181648	8	.9496	.9518	2.466323	8	.9895	.9903	2.988611	8
25	20	.8893	.9009	2.171764	7	.9496	.9559	2.487461	8	.9900	.9911	3.061861	8
30	30	.8987	.9211	2.335496	8	.9479	.9560	2.581983	9	.9898	.9901	3.098385	9
30	29	.8982	.9022	2.277955	8	.9481	.9537	2.540984	8	.9899	.9903	3.072931	9
30	28	.8914	.9032	2.259842	8	.9496	.9515	2.545241	8	.9893	.9902	3.101778	8
30	27	.8996	.9047	2.250919	8	.9483	.9500	2.515931	8	.9895	.9901	3.071769	8
30	26	.8998	.9020	2.243121	8	.9492	.9506	2.548161	8	.9888	.9900	3.046899	9
30	25	.8969	.9017	2.262552	7	.9500	.9523	2.542743	8	.9897	.9902	3.065360	9
35	35	.8985	.9021	2.298919	8	.9464	.9516	2.561732	8	.9897	.9902	3.112678	9
35	34	.8911	.9022	2.288361	8	.9494	.9510	2.563600	8	.9898	.9901	3.107550	9
35	33	.8994	.9010	2.262083	8	.9492	.9503	2.530571	8	.9898	.9905	3.125223	8
35	32	.8996	.9007	2.294444	8	.9491	.9501	2.562941	8	.9898	.9906	3.083079	9
35	31	.8989	.9008	2.267177	8	.9483	.9503	2.546338	8	.9897	.9906	3.071009	9
35	30	.8996	.9012	2.281705	7	.9494	.9506	2.550909	8	.9899	.9903	3.100878	9
40	40	.8929	.9060	2.309393	8	.9491	.9516	2.581981	8	.9896	.9902	3.135714	9
40	39	.8997	.9022	2.305096	8	.9497	.9504	2.594047	8	.9898	.9901	3.120332	9
40	38	.8991	.9006	2.303047	8	.9484	.9512	2.588710	8	.9895	.9902	3.108892	9
40	37	.8989	.9001	2.323151	7	.9495	.9502	2.602662	8	.9895	.9902	3.105713	9
40	36	.8982	.9000	2.319509	7	.9495	.9504	2.574150	8	.9899	.9902	3.143799	9
40	35	.8980	.9059	2.314550	8	.9500	.9536	2.599129	8	.9899	.9901	3.105164	9
45	45	.9000	.9026	2.325646	8	.9489	.9507	2.588727	8	.9898	.9908	3.162276	9
45	44	.8985	.9006	2.305894	8	.9494	.9502	2.606466	8	.9896	.9902	3.128894	9
45	43	.8999	.9013	2.339338	7	.9487	.9507	2.611434	8	.9899	.9902	3.133795	9
45	42	.8993	.9003	2.312020	8	.9492	.9503	2.598805	8	.9898	.9900	3.147068	9
45	41	.8997	.9007	2.338587	8	.9495	.9502	2.581439	9	.9898	.9903	3.132746	9
45	40	.8985	.9003	2.305622	8	.9491	.9525	2.603961	8	.9899	.9901	3.124396	9
50	50	.8998	.9025	2.341460	8	.9494	.9504	2.625856	8	.9891	.9900	3.144848	9
50	49	.8974	.9012	2.345972	8	.9487	.9506	2.634603	8	.9899	.9904	3.149229	9
50	48	.8997	.9006	2.364355	8	.9497	.9504	2.624764	9	.9899	.9901	3.174269	9
50	47	.8973	.9008	2.353458	8	.9497	.9503	2.623077	9	.9900	.9901	3.160010	9
50	46	.8959	.9001	2.321761	8	.9498	.9503	2.635529	8	.9896	.9902	3.139391	9
50	45	.8985	.9026	2.345725	8	.9480	.9514	2.607892	9	.9899	.9901	3.161185	9
100	100	.9000	.9003	2.404278	9	.9498	.9501	2.693607	9	.9900	.9901	3.256759	9
500	500	.9000	.9001	2.440191	[illegible]	.9500	.9500	2.743759	9	.9900	.9900	3.326801	9

$\theta = .25$ $\sqrt{MN/(M+N)}\,\tilde{W}^{+}_{M,N}$ 2 - 10

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
2	2	.8333	1	1.999992	7	.8333	1	1.999992	7	.8333	1	1.999992	7
3	3	.8000	1	1.732044	6	.8000	1	1.732044	6	.8000	1	1.732044	6
3	2	.7000	1	1.490709	9	.7000	1	1.490709	9	.7000	1	1.490709	9
4	4	.7857	.9286	1.632990	9	.9286	1	2.190887	6	.9286	1	2.190887	6
4	3	.8857	1	1.984305	8	.8857	1	1.984305	8	.8857	1	1.984305	8
4	2	.8000	1	1.732044	6	.8000	1	1.732044	6	.8000	1	1.732044	6
5	5	.8571	.9167	1.897363	5	.9167	.9762	2.070193	9	.9762	1	2.581984	7
5	4	.8810	.9603	1.897366	5	.8810	.9603	1.897366	5	.9603	1	2.399993	8
5	3	.8214	.9286	1.697055	7	.9286	1	2.190886	5	.9286	1	2.190886	5
5	2	.8571	.9990	1.932183	8	.8571	1	1.932183	8	.8571	1	1.932183	8
6	6	.8896	.9437	1.999996	5	.9437	.9697	2.309397	9	.9697	.9924	2.449480	9
6	5	.8463	.9113	1.854047	6	.9113	.9545	2.100528	5	.9870	1	2.763848	9
6	4	.8524	.9286	1.844657	7	.9286	.9762	2.108181	6	.9762	1	2.581984	5
6	3	.8810	.9524	1.897361	9	.8810	.9524	1.897361	9	.9524	1	2.371701	7
6	2	.8929	1	2.108181	5	.8929	1	2.108181	5	.8929	1	2.108181	5
7	7	.8794	.9324	1.954012	6	.9324	.9650	2.160243	6	.9895	1	2.788861	9
7	6	.8409	.9021	1.828345	7	.9021	.9510	1.974462	7	.9837	1	2.638991	5
7	5	.8535	.9293	1.756613	9	.9293	.9735	2.070189	7	.9735	1	2.474358	6
7	4	.8939	.9545	1.895211	9	.8939	.9545	1.895211	9	.9545	1	2.288685	8
7	3	.8333	.9167	1.690308	6	.9167	1	2.070193	5	.9167	1	2.070193	5
7	2	.8333	1	1.792838	8	.8333	1	1.792838	8	.8333	1	1.792838	8
8	8	.8938	.9242	1.999992	7	.9242	.9503	2.065590	7	.9872	.9965	2.696795	5
8	7	.8965	.9313	1.901588	8	.9313	.9596	2.184653	7	.9814	.9944	2.561731	6
8	6	.8555	.9021	1.714925	5	.9371	.9720	2.160239	8	.9720	.9907	2.415222	7
8	5	.8982	.9565	1.935261	5	.8982	.9565	1.935261	5	.9837	1	2.638987	7
8	4	.8586	.9293	1.732044	6	.9293	.9697	2.070193	5	.9697	1	2.449480	9
8	3	.8788	.9394	1.854042	7	.9394	1	2.224858	6	.9394	1	2.224858	6
8	2	.8667	1	1.936482	9	.8667	1	1.936482	9	.8667	1	1.936482	9
9	9	.8121	.9077	1.897361	9	.9319	.9535	2.267783	7	.9899	.9955	2.846046	6
9	8	.8766	.9066	1.854160	9	.9319	.9550	2.176080	8	.9843	.9932	2.691467	6
9	7	.8684	.9014	1.968252	9	.9344	.9589	2.095237	9	.9895	.9969	2.732519	7
9	6	.8909	.9371	1.906921	5	.9371	.9580	2.236064	9	.9832	.9944	2.581984	8
9	5	.8996	.9271	2.078693	5	.9271	.9720	2.093198	5	.9895	1	2.788864	8
9	4	.8322	.9021	1.805277	7	.9021	.9510	1.900286	7	.9790	1	2.596293	5
9	3	.8409	.9091	1.690305	5	.9091	.9545	1.999996	8	.9545	1	2.366426	7
9	2	.8909	1	2.068277	5	.8909	1	2.068277	5	.8909	1	2.068277	5
10	10	.8887	.9115	1.951796	9	.9319	.9595	2.247325	8	.9874	.9928	2.738606	6
10	9	.8798	.9010	2.057129	9	.9437	.9597	2.205804	9	.9893	.9942	2.809325	7
10	8	.8897	.9201	2.028367	5	.9409	.9569	2.333446	9	.9838	.9905	2.683280	7
10	7	.8707	.9107	1.913509	6	.9377	.9571	2.226730	5	.9838	.9938	2.671569	8
10	6	.8605	.9073	1.866660	6	.9292	.9580	2.088931	6	.9895	.9965	2.732514	9
10	5	.8911	.9281	1.936484	7	.9281	.9814	2.236064	6	.9814	.9930	2.561732	5
10	4	.8701	.9301	1.940216	8	.9301	.9650	2.049389	8	.9850	1	2.732519	6
10	3	.8776	.9301	1.828347	5	.9301	.9650	2.133068	9	.9650	1	2.497991	8
10	2	.8485	.9091	1.833026	7	.9091	1	2.190889	6	.9091	1	2.190889	6

θ = .25 (continued) $\sqrt{MN/(M+N)}\;\tilde{W}^{+}_{M,N}$ 15 - 500

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
15	15	.8899	.9104	2.148342	6	.9474	.9540	2.323785	8	.9874	.9904	2.788863	8
15	14	.8974	.9232	2.081021	7	.9409	.9519	2.374630	7	.9865	.9901	2.795149	8
15	13	.8843	.9130	2.011077	7	.9416	.9558	2.296813	7	.9896	.9924	2.844091	7
15	12	.8983	.9146	2.078457	7	.9341	.9508	2.215639	8	.9889	.9921	2.762508	7
15	11	.8882	.9015	1.984786	7	.9429	.9546	2.327569	7	.9880	.9928	2.778701	8
15	10	.8970	.9194	2.041237	7	.9472	.9566	2.288019	7	.9892	.9923	2.796294	8
20	20	.8936	.9053	2.080623	7	.9452	.9561	2.390451	7	.9884	.9903	2.912872	8
20	19	.8913	.9035	2.087496	7	.9479	.9506	2.399391	7	.9881	.9907	2.846872	8
20	18	.8966	.9027	2.064013	7	.9447	.9501	2.299964	7	.9889	.9908	2.910970	7
20	17	.8948	.9016	2.111073	7	.9497	.9556	2.429547	7	.9893	.9906	2.897456	8
20	16	.8996	.9068	2.103502	7	.9476	.9520	2.371703	8	.9888	.9908	2.868548	8
20	15	.8813	.9178	2.091644	7	.9427	.9524	2.330456	8	.9886	.9915	2.840562	8
25	25	.8981	.9022	2.089784	7	.9495	.9521	2.425355	7	.9897	.9906	2.946277	8
25	24	.8995	.9028	2.146663	7	.9489	.9542	2.444417	8	.9891	.9903	3.001710	8
25	23	.8967	.9017	2.099480	8	.9487	.9525	2.452055	7	.9895	.9904	2.948499	8
25	22	.8967	.9037	2.149519	7	.9461	.9503	2.446064	8	.9900	.9910	2.915005	8
25	21	.8967	.9013	2.072035	7	.9473	.9550	2.407483	7	.9892	.9901	2.919569	8
25	20	.8985	.9070	2.131319	7	.9484	.9556	2.449484	7	.9892	.9902	2.939728	8
30	30	.8974	.9077	2.143197	8	.9487	.9505	2.435992	8	.9895	.9906	3.038212	8
30	29	.8972	.9000	2.176025	8	.9469	.9507	2.479770	8	.9896	.9901	2.994507	8
30	28	.8983	.9012	2.125330	8	.9458	.9501	2.427483	7	.9886	.9900	2.943007	8
30	27	.8910	.9004	2.130333	7	.9483	.9505	2.473093	7	.9892	.9900	3.025780	8
30	26	.8959	.9022	2.095519	8	.9485	.9513	2.398618	7	.9898	.9906	2.985170	8
30	25	.8993	.9022	2.134530	8	.9473	.9508	2.482947	7	.9898	.9914	2.994556	8
35	35	.8967	.9012	2.151483	8	.9490	.9509	2.418963	8	.9894	.9902	2.988069	8
35	34	.8994	.9045	2.097262	8	.9486	.9500	2.439960	7	.9898	.9902	3.018046	8
35	33	.8976	.9008	2.153479	8	.9469	.9508	2.428471	8	.9899	.9903	3.026474	8
35	32	.8988	.9013	2.097488	8	.9491	.9508	2.433118	7	.9894	.9900	3.030095	8
35	31	.8975	.9001	2.126106	8	.9486	.9506	2.419558	8	.9899	.9904	3.003704	8
35	30	.8989	.9037	2.095047	8	.9491	.9506	2.418671	7	.9894	.9904	2.992335	8
40	40	.8974	.9035	2.121453	8	.9500	.9521	2.479114	7	.9896	.9901	3.006366	8
40	39	.8998	.9017	2.148612	8	.9489	.9503	2.440239	8	.9897	.9901	3.044987	8
40	38	.8998	.9017	2.113754	8	.9496	.9518	2.475986	7	.9898	.9901	3.034829	8
40	37	.8994	.9012	2.149215	8	.9488	.9501	2.422312	8	.9898	.9903	3.044672	8
40	36	.8985	.9012	2.122190	8	.9496	.9508	2.448950	7	.9896	.9901	2.986234	8
40	35	.8978	.9007	2.122915	8	.9492	.9504	2.424494	7	.9897	.9900	3.039490	8
45	45	.8995	.9027	2.151651	8	.9497	.9510	2.452755	8	.9898	.9901	3.043797	8
45	44	.8997	.9031	2.158836	8	.9496	.9508	2.472516	8	.9899	.9901	3.073079	8
45	43	.8985	.9002	2.143591	8	.9483	.9505	2.455378	8	.9900	.9903	3.027972	8
45	42	.8999	.9023	2.166723	8	.9486	.9515	2.470125	8	.9898	.9901	3.064728	8
45	41	.8998	.9018	2.138025	8	.9495	.9504	2.459195	7	.9899	.9903	3.021163	8
45	40	.8990	.9006	2.133925	8	.9497	.9507	2.462650	7	.9898	.9901	3.052642	8
50	50	.8987	.9044	2.162175	8	.9496	.9506	2.477698	8	.9898	.9905	3.075342	8
50	49	.8990	.9006	2.146054	8	.9476	.9502	2.480402	7	.9899	.9901	3.079273	8
50	48	.8987	.9000	2.135232	8	.9489	.9500	2.449984	7	.9898	.9901	3.062879	8
50	47	.8982	.9032	2.148523	8	.9485	.9503	2.467554	8	.9900	.9902	3.058764	8
50	46	.8990	.9002	2.175885	8	.9473	.9503	2.438462	8	.9899	.9901	3.059521	8
50	45	.8997	.9011	2.142386	8	.9494	.9502	2.491106	7	.9899	.9901	3.078716	8
100	100	.8996	.9001	2.160246	5	.9495	.9501	2.483003	8	.9897	.9901	3.103203	8
500	500	.9000	.9001	2.192262	6	.9500	.9500	2.522723	8	.9900	.9900	3.137082	8

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
See $W_{M,N}$ for θ = 0.													
θ = .01 (see θ = 0 for smaller values of M)													
100	100	.8984	.9010	2.789056	8	.9495	.9502	3.011905	5	.9899	.9900	3.496298	5
500	500	.8996	.9007	2.926527	5	.9495	.9503	3.178208	5	.9900	.9900	3.662334	5
θ = .05 (see θ = 0 for smaller values of M)													
10	10	.8813	.9343	2.344035	8	.9343	.9522	2.581984	7	.9893	.9961	3.146266	6
10	9	.8731	.9034	2.205804	9	.9376	.9584	2.556737	7	.9884	.9939	3.002796	6
10	8	.8701	.9058	2.347866	9	.9358	.9569	2.439517	8	.9899	.9950	3.027148	7
10	7	.8976	.9284	2.265026	5	.9459	.9675	2.608386	8	.9826	.9905	2.886168	8
10	6	.8749	.9161	2.367453	5	.9423	.9685	2.472279	5	.9810	.9930	2.933331	8
10	5	.8561	.9394	2.236064	6	.9394	.9594	2.561732	5	.9860	.9960	2.927695	9
10	4	.8741	.9061	2.366431	7	.9061	.9700	2.432075	6	.9700	.9900	2.732519	6
10	3	.8601	.9301	2.133068	9	.9301	.9720	2.497991	8	.9720	1	2.962258	6
10	2	.8182	.9091	2.190889	6	.9091	1	2.683280	5	.9091	1	2.683280	5
15	15	.8925	.9191	2.477166	5	.9373	.9504	2.652067	9	.9861	.9914	3.098386	9
15	14	.8917	.9007	2.419673	7	.9406	.9505	2.657223	9	.9900	.9916	3.195054	9
15	13	.8823	.9041	2.406539	8	.9467	.9542	2.654931	9	.9866	.9902	3.100610	9
15	12	.8826	.9031	2.464744	7	.9455	.9535	2.598076	5	.9878	.9904	3.065335	9
15	11	.8968	.9161	2.391645	7	.9497	.9565	2.688571	9	.9859	.9901	3.109498	9
15	10	.8923	.9160	2.485337	7	.9440	.9502	2.545875	5	.9900	.9926	3.105167	8
20	20	.8987	.9070	2.542563	7	.9476	.9553	2.760259	5	.9872	.9903	3.186959	5
20	19	.8967	.9033	2.491825	7	.9449	.9516	2.749011	9	.9899	.9909	3.219319	5
20	18	.8901	.9021	2.532286	7	.9491	.9532	2.757078	9	.9899	.9909	3.253447	5
20	17	.8868	.9007	2.467197	8	.9475	.9512	2.745368	9	.9889	.9902	3.180658	5
20	16	.8833	.9016	2.448279	8	.9462	.9505	2.683274	9	.9891	.9904	3.174892	9
20	15	.8930	.9003	2.484279	7	.9493	.9610	2.788862	9	.9899	.9909	3.231544	5
25	25	.8874	.9015	2.576729	8	.9432	.9503	2.828423	5	.9899	.9915	3.311327	5
25	24	.8955	.9062	2.561975	8	.9487	.9538	2.799997	5	.9896	.9901	3.283810	5
25	23	.8982	.9040	2.563553	8	.9472	.9504	2.749647	5	.9893	.9906	3.250980	9
25	22	.8942	.9057	2.532098	8	.9484	.9507	2.787050	9	.9898	.9904	3.323183	5
25	21	.8965	.9009	2.586718	8	.9479	.9509	2.790875	5	.9892	.9901	3.253275	5
25	20	.8972	.9011	2.533955	8	.9489	.9532	2.789934	9	.9900	.9914	3.287308	5
30	30	.8868	.9096	2.581985	8	.9482	.9545	2.820314	5	.9898	.9909	3.286335	5
30	29	.8892	.9002	2.540980	8	.9496	.9512	2.786832	9	.9897	.9902	3.288573	5
30	28	.8996	.9023	2.573542	8	.9479	.9501	2.777412	5	.9899	.9904	3.278530	5
30	27	.8916	.9002	2.547258	8	.9498	.9535	2.813650	9	.9896	.9901	3.301188	5
30	26	.8971	.9013	2.561233	8	.9446	.9517	2.784569	5	.9892	.9900	3.248184	5
30	25	.8957	.9000	2.542741	8	.9500	.9540	2.793098	9	.9897	.9905	3.316624	5
35	35	.8986	.9048	2.621583	8	.9469	.9505	2.846371	5	.9899	.9905	3.359000	5
35	34	.8973	.9001	2.583537	8	.9478	.9514	2.829075	5	.9898	.9903	3.328848	5
35	33	.8984	.9023	2.639676	8	.9496	.9507	2.867230	5	.9895	.9901	3.324781	5
35	32	.8995	.9034	2.594921	8	.9496	.9518	2.837435	5	.9898	.9904	3.305329	5
35	31	.8983	.9017	2.581086	8	.9492	.9512	2.828540	5	.9897	.9903	3.309601	5
35	30	.8935	.9008	2.593088	8	.9498	.9511	2.832462	5	.9897	.9901	3.322165	5
40	40	.8998	.9028	2.653298	8	.9479	.9503	2.889983	5	.9897	.9901	3.355147	5
40	39	.8987	.9001	2.629184	8	.9488	.9500	2.869374	5	.9896	.9901	3.341112	5
40	38	.8986	.9026	2.623474	8	.9495	.9504	2.872474	5	.9897	.9902	3.350037	5
40	37	.8989	.9014	2.635697	8	.9489	.9520	2.873666	5	.9896	.9903	3.335859	5
40	36	.8985	.9000	2.610223	8	.9499	.9512	2.855678	5	.9896	.9900	3.337409	5
40	35	.8988	.9004	2.603861	8	.9499	.9508	2.872336	5	.9899	.9902	3.339556	5
45	45	.8980	.9011	2.658352	8	.9489	.9527	2.908374	5	.9900	.9904	3.380613	5
45	44	.8981	.9010	2.657536	8	.9492	.9502	2.888121	5	.9899	.9902	3.368466	5
45	43	.8987	.9000	2.644794	8	.9494	.9506	2.871076	5	.9898	.9901	3.405520	5
45	42	.8996	.9005	2.663692	8	.9494	.9500	2.879867	5	.9899	.9901	3.359984	5
45	41	.8963	.9027	2.634930	8	.9493	.9502	2.891464	5	.9899	.9900	3.366925	5
45	40	.8995	.9015	2.661453	8	.9494	.9502	2.894621	5	.9899	.9902	3.370232	5

θ = 0.05 / .1 $\sqrt{MN/(M+N)}\,\tilde{W}_{M,N}$ 50 - 500 / 5 - 9

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
						θ = .05 (continued)							
50	50	.8991	.9025	2.666661	8	.9489	.9508	2.896826	5	.9897	.9902	3.379627	5
50	49	.8998	.9021	2.653292	8	.9499	.9503	2.914943	5	.9900	.9901	3.377394	5
50	48	.8998	.9021	2.644345	8	.9484	.9519	2.891825	5	.9899	.9901	3.385744	5
50	47	.8966	.9010	2.658960	8	.9495	.9507	2.873315	5	.9899	.9901	3.366967	5
50	46	.8941	.9004	2.635620	8	.9495	.9503	2.884155	5	.9900	.9901	3.360853	5
50	45	.8998	.9014	2.640611	8	.9497	.9514	2.897347	5	.9900	.9901	3.375884	5
100	100	.8997	.9003	2.736859	8	.9497	.9501	2.981947	5	.9899	.9900	3.472314	5
500	500	.9000	.9000	2.836107	5	.9499	.9500	3.098615	5	.9900	.9900	3.610451	5

θ = .1 (see θ = 0 for smaller values of M)

5	5	.8333	.9524	2.070193	9	.8333	.9524	2.070193	9	.9524	1	2.581984	7
5	4	.7619	.9206	1.897366	5	.9206	1	2.399993	8	.9206	1	2.399993	8
5	3	.8571	1	2.190886	5	.8571	1	2.190886	5	.8571	1	2.190886	5
5	2	.7143	1	1.932183	8	.7143	1	1.932183	8	.7143	1	1.932183	8
6	6	.8874	.9394	2.309397	9	.9394	.9848	2.449480	9	.9848	1	2.927695	7
6	5	.8225	.9091	2.100528	5	.9091	.9740	2.288684	5	.9740	1	2.763848	9
6	4	.8571	.9524	2.108181	6	.8571	.9524	2.108181	6	.9524	1	2.581984	5
6	3	.7619	.9048	1.897361	9	.9048	1	2.371701	7	.9048	1	2.371701	7
6	2	.7857	1	2.108181	5	.7857	1	2.108181	5	.7857	1	2.108181	5
7	7	.8159	.9138	2.160243	6	.9138	.9586	2.366431	5	.9790	.9953	2.788861	9
7	6	.8741	.9324	2.225394	6	.9324	.9674	2.489542	5	.9674	.9918	2.638991	5
7	5	.8838	.9470	2.276409	7	.9470	.9848	2.474358	6	.9848	1	2.927695	5
7	4	.7879	.9091	2.013658	9	.9091	.9697	2.288685	8	.9697	1	2.746424	6
7	3	.8333	.9333	2.070193	5	.9333	1	2.535457	8	.9333	1	2.535457	8
7	2	.8333	1	2.267786	6	.8333	1	2.267786	6	.8333	1	2.267786	6
8	8	.8812	.9246	2.309397	6	.9246	.9681	2.519758	6	.9855	.9930	2.999995	5
8	7	.8819	.9192	2.324172	7	.9192	.9518	2.351452	6	.9888	.9975	2.958038	5
8	6	.8741	.9241	2.160239	8	.9241	.9574	2.415222	7	.9814	.9953	2.806237	6
8	5	.8741	.9207	2.253461	8	.9207	.9674	2.433745	8	.9674	.9907	2.638987	7
8	4	.8424	.9394	2.165058	5	.9394	.9798	2.449480	9	.9798	1	2.898269	7
8	3	.8788	.9515	2.224858	6	.8788	.9515	2.224858	6	.9515	1	2.686768	5
8	2	.8667	1	2.415227	7	.8667	1	2.415227	7	.8667	1	2.415227	7
9	9	.8854	.9027	2.357021	7	.9027	.9531	2.417466	7	.9887	.9951	2.999994	5
9	8	.8984	.9301	2.265026	8	.9301	.9531	2.509241	7	.9825	.9917	2.870957	6
9	7	.8916	.9213	2.378350	8	.9497	.9717	2.519761	7	.9851	.9937	2.984126	6
9	6	.8242	.9161	2.236064	9	.9161	.9520	2.324167	9	.9888	.9972	2.958034	7
9	5	.8541	.9141	2.093198	5	.9441	.9790	2.577578	9	.9790	.9940	2.788864	8
9	4	.8797	.9580	2.303545	6	.8797	.9580	2.303545	6	.9860	1	3.040460	9
9	3	.8182	.9091	1.999996	8	.9091	.9636	2.366426	7	.9636	1	2.828422	6
9	2	.8909	1	2.553138	9	.8909	1	2.553138	9	.8909	1	2.553138	9

$\theta = .1$ (continued) $\sqrt{MN/(M+N)}\,\tilde{W}_{M,N}$ 10 - 40

M	N	$P(\underline{z}_{.9})$	$P(\bar{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\bar{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\bar{z}_{.99})$	$\underline{z}_{.99}$	D
10	10	.8813	.9343	2.344035	8	.9343	.9522	2.581984	7	.9893	.9961	3.146266	6
10	9	.8731	.9034	2.205804	9	.9376	.9584	2.556737	7	.9884	.9939	3.002796	6
10	8	.8701	.9058	2.347866	9	.9358	.9569	2.439517	8	.9899	.9950	3.027148	7
10	7	.8976	.9284	2.265026	5	.9459	.9675	2.608386	8	.9826	.9905	2.886168	8
10	6	.8749	.9161	2.367453	5	.9423	.9685	2.472279	5	.9810	.9930	2.933331	8
10	5	.8561	.9394	2.236064	6	.9394	.9594	2.561732	5	.9860	.9960	2.927695	9
10	4	.8741	.9061	2.366431	7	.9061	.9700	2.432075	6	.9700	1	2.732519	6
10	3	.8601	.9301	2.133068	9	.9301	.9720	2.497991	8	.9720	1	2.962258	6
10	2	.8182	.9091	2.190889	6	.9091	1	2.683280	5	.9091	1	2.683280	5
15	15	.8930	.9119	2.449489	5	.9424	.9592	2.652069	9	.9898	.9922	3.098378	9
15	14	.8998	.9095	2.419675	7	.9440	.9540	2.657224	9	.9897	.9923	3.174198	9
15	13	.8967	.9102	2.306544	8	.9477	.9539	2.652520	8	.9885	.9911	3.066356	9
15	12	.8952	.9237	2.464748	7	.9449	.9502	2.497636	5	.9895	.9922	3.065332	8
15	11	.8920	.9057	2.342722	7	.9435	.9529	2.650380	8	.9869	.9909	3.075460	9
15	10	.8767	.9008	2.294152	8	.9441	.9566	2.545867	8	.9894	.9920	3.061858	9
20	20	.8865	.9047	2.478742	7	.9404	.9502	2.656844	9	.9893	.9907	3.186955	9
20	19	.8952	.9009	2.471597	7	.9471	.9501	2.728069	9	.9893	.9914	3.171416	5
20	18	.8969	.9046	2.493681	7	.9444	.9521	2.714713	9	.9896	.9905	3.248922	9
20	17	.8938	.9078	2.467200	8	.9434	.9502	2.708933	9	.9897	.9910	3.179445	9
20	16	.8900	.9067	2.399992	8	.9458	.9561	2.662558	9	.9899	.9912	3.174892	9
20	15	.8876	.9047	2.456168	8	.9465	.9505	2.732840	9	.9899	.9915	3.155242	5
25	25	.8982	.9005	2.545583	7	.9456	.9526	2.777459	5	.9885	.9902	3.204940	5
25	24	.8999	.9036	2.525614	7	.9490	.9526	2.763904	9	.9897	.9908	3.253304	5
25	23	.8974	.9016	2.504570	8	.9490	.9548	2.745811	9	.9900	.9908	3.213438	5
25	22	.8967	.9007	2.475997	8	.9477	.9502	2.750869	9	.9898	.9908	3.170087	5
25	21	.8992	.9037	2.466327	7	.9490	.9524	2.675115	9	.9898	.9906	3.244110	9
25	20	.8993	.9119	2.487467	8	.9495	.9526	2.757707	9	.9896	.9905	3.203243	5
30	30	.8960	.9120	2.581987	8	.9494	.9532	2.817180	5	.9895	.9901	3.279564	5
30	29	.8964	.9074	2.540978	8	.9480	.9533	2.770841	5	.9896	.9901	3.262785	5
30	28	.8994	.9012	2.545237	8	.9497	.9519	2.777407	5	.9896	.9902	3.262735	5
30	27	.8967	.9002	2.515930	8	.9495	.9511	2.792786	9	.9898	.9904	3.293592	5
30	26	.8986	.9013	2.548157	8	.9473	.9564	2.784569	5	.9898	.9906	3.248184	5
30	25	.8971	.9000	2.533617	8	.9498	.9521	2.780729	9	.9895	.9901	3.226618	5
35	35	.8931	.9033	2.561736	8	.9491	.9521	2.799167	5	.9895	.9903	3.308106	5
35	34	.8989	.9022	2.563604	8	.9498	.9510	2.808865	5	.9897	.9901	3.294606	5
35	33	.8985	.9007	2.530567	8	.9493	.9511	2.819228	9	.9898	.9901	3.297518	5
35	32	.8984	.9004	2.562937	8	.9487	.9512	2.808550	5	.9897	.9901	3.288755	5
35	31	.8966	.9007	2.546339	8	.9495	.9509	2.793386	9	.9895	.9901	3.249343	5
35	30	.8990	.9012	2.550915	7	.9452	.9502	2.796340	9	.9898	.9902	3.305529	5
40	40	.8982	.9033	2.581983	8	.9497	.9517	2.864459	9	.9894	.9901	3.341867	5
40	39	.8995	.9009	2.594046	8	.9499	.9508	2.844281	5	.9898	.9901	3.326882	5
40	38	.8970	.9025	2.588708	8	.9495	.9518	2.837070	5	.9897	.9902	3.301735	5
40	37	.8991	.9004	2.602679	8	.9495	.9510	2.842478	5	.9898	.9905	3.323741	5
40	36	.8992	.9010	2.574154	8	.9492	.9507	2.826069	5	.9899	.9903	3.332631	5
40	35	.8989	.9001	2.598192	8	.9493	.9511	2.834733	5	.9898	.9902	3.314846	5

θ = 0.1 / .25 $\sqrt{MN/(M+N)}\,\tilde{W}_{M,N}$ 45 -500 / 2 - 9

M	N	$P(\underline{z}_{.9})$	$P(\tilde{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\tilde{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\tilde{z}_{.99})$	$\underline{z}_{.99}$	D
						θ = .1 (continued)							
45	45	.8979	.9016	2.588728	8	.9495	.9517	2.860383	5	.9899	.9904	3.354102	5
45	44	.8989	.9007	2.606463	8	.9493	.9504	2.865445	5	.9895	.9901	3.318955	5
45	43	.8977	.9016	2.611430	8	.9497	.9506	2.852728	5	.9899	.9902	3.326269	5
45	42	.8986	.9007	2.598801	8	.9481	.9511	2.867537	5	.9899	.9902	3.351490	5
45	41	.8993	.9006	2.581444	8	.9494	.9502	2.829832	5	.9898	.9903	3.318489	5
45	40	.8983	.9052	2.603958	8	.9498	.9506	2.857039	5	.9898	.9901	3.331101	5
50	50	.8990	.9010	2.625854	8	.9495	.9511	2.881951	5	.9896	.9900	3.377599	5
50	49	.8977	.9014	2.634608	8	.9498	.9506	2.885253	5	.9895	.9900	3.340310	5
50	48	.8995	.9009	2.624761	8	.9498	.9508	2.867625	5	.9900	.9903	3.376226	5
50	47	.8997	.9009	2.623080	8	.9478	.9508	2.862856	5	.9900	.9902	3.357593	5
50	46	.8998	.9008	2.635529	8	.9492	.9505	2.861924	5	.9895	.9901	3.340571	5
50	45	.8963	.9030	2.607892	8	.9493	.9500	2.861921	5	.9897	.9901	3.355481	5
100	100	.8998	.9004	2.693605	8	.9500	.9512	2.964997	5	.9900	.9901	3.459950	5
500	500	.9000	.9000	2.742831	5	.9500	.9500	3.012909	5	.9900	.9900	3.545579	5
						θ = .25							
2	2	.6667	1	1.999992	7	.6667	1	1.999992	7	.6667	1	1.999992	7
3	3	.6000	1	1.732044	6	.6000	1	1.732044	6	.6000	1	1.732044	6
3	2	.4000	1	1.490709	9	.4000	1	1.490709	9	.4000	1	1.490709	9
4	4	.8571	1	2.190887	6	.8571	1	2.190887	6	.8571	1	2.190887	6
4	3	.7714	1	1.984305	8	.7714	1	1.984305	6	.7714	1	1.984305	8
4	2	.6000	1	1.732044	6	.6000	1	1.732044	6	.6000	1	1.732044	6
5	5	.8333	.9524	2.070193	9	.8333	.9524	2.070193	9	.9524	1	2.581984	7
5	4	.7619	.9206	1.897366	5	.9206	1	2.399993	8	.9206	1	2.399993	8
5	3	.8571	1	2.190886	5	.8571	1	2.190886	5	.8571	1	2.190886	5
5	2	.7143	1	1.932183	8	.7143	1	1.932183	8	.7143	1	1.932183	8
6	6	.8874	.9394	2.309397	9	.9394	.9848	2.449480	9	.9848	1	2.927695	7
6	5	.8225	.9091	2.100528	5	.9091	.9740	2.288684	5	.9740	1	2.763848	9
6	4	.8571	.9524	2.108181	6	.8571	.9524	2.108181	6	.9524	1	2.581984	5
6	3	.7619	.9048	1.897361	9	.9048	1	2.371701	7	.9048	1	2.371701	7
6	2	.7857	1	2.108181	5	.7857	1	2.108181	5	.7857	1	2.108181	5
7	7	.8648	.9301	2.160243	6	.9301	.9790	2.366431	5	.9790	1	2.788861	9
7	6	.8042	.9021	1.974462	7	.9021	.9674	2.225394	6	.9674	1	2.638991	5
7	5	.8586	.9470	2.070189	7	.9470	1	2.474358	6	.9470	1	2.474358	6
7	4	.7879	.9091	1.895211	9	.9091	1	2.288685	8	.9091	1	2.288685	8
7	3	.8333	1	2.070193	5	.8333	1	2.070193	5	.8333	1	2.070193	5
7	2	.6667	1	1.792838	8	.6667	1	1.792838	8	.6667	1	1.792838	8
8	8	.8486	.9008	2.065590	7	.9464	.9744	2.519758	6	.9744	.9930	2.696795	5
8	7	.8628	.9192	2.184653	7	.9192	.9627	2.351452	6	.9888	1	2.958038	5
8	6	.8741	.9441	2.160239	8	.9441	.9814	2.415222	7	.9814	1	2.806237	6
8	5	.7964	.9130	1.935261	5	.9130	.9674	2.253461	8	.9674	1	2.638987	7
8	4	.8586	.9394	2.070193	5	.9394	1	2.449480	9	.9394	1	2.449480	9
8	3	.8788	1	2.224858	6	.8788	1	2.224858	6	.8788	1	2.224858	6
8	2	.7333	1	1.936482	9	.7333	1	1.936482	9	.7333	1	1.936482	9
9	9	.8640	.9070	2.267783	7	.9367	.9613	2.417466	7	.9798	.9910	2.846046	6
9	8	.8638	.9099	2.176080	8	.9445	.9687	2.509241	7	.9864	.9963	2.870957	6
9	7	.8689	.9178	2.095237	9	.9497	.9790	2.519761	7	.9790	.9937	2.732519	7
9	6	.8741	.9161	2.236064	9	.9161	.9664	2.324167	9	.9888	1	2.958034	7
9	5	.8541	.9441	2.093198	5	.9441	.9790	2.415220	9	.9790	1	2.788864	8
9	4	.8042	.9021	1.900286	7	.9021	.9580	2.225390	6	.9580	1	2.596293	5
9	3	.8182	.9091	1.999996	8	.9091	1	2.366426	7	.9091	1	2.366426	7
9	2	.7818	1	2.068277	5	.7818	1	2.068277	5	.7818	1	2.068277	5

$\theta = 0.25$ (continued) $\sqrt{MN/(M+N)}\,\tilde{W}_{M,N}$ 10 - 500

M	N	$P(\underline{z}_{.9})$	$P(\tilde{z}_{.9})$	$\underline{z}_{.9}$	D	$P(\underline{z}_{.95})$	$P(\tilde{z}_{.95})$	$\underline{z}_{.95}$	D	$P(\underline{z}_{.99})$	$P(\tilde{z}_{.99})$	$\underline{z}_{.99}$	D
10	10	.8638	.9191	2.247325	8	.9434	.9617	2.581984	7	.9856	.9927	2.927695	6
10	9	.8875	.9193	2.205804	9	.9420	.9631	2.518472	7	.9884	.9952	3.002796	6
10	8	.8818	.9138	2.333446	9	.9445	.9675	2.439517	8	.9809	.9925	2.846045	7
10	7	.8754	.9142	2.226730	5	.9488	.9675	2.547718	9	.9877	.9963	2.886168	8
10	6	.8585	.9161	2.088931	6	.9423	.9790	2.472279	5	.9790	.9930	2.732514	9
10	5	.8561	.9627	2.236064	6	.8561	.9627	2.236064	6	.9860	1	2.927695	9
10	4	.8601	.9301	2.049389	8	.9301	.9700	2.366431	7	.9700	1	2.732519	6
10	3	.8601	.9301	2.133068	9	.9301	1	2.497991	8	.9301	1	2.497991	8
10	2	.8182	1	2.190889	6	.8182	1	2.190889	6	.8182	1	2.190889	6
15	15	.8949	.9082	2.323786	6	.9398	.9507	2.556032	9	.9889	.9925	3.021657	9
15	14	.8821	.9038	2.374627	7	.9459	.9553	2.535284	9	.9893	.9928	2.992506	9
15	13	.8834	.9116	2.296810	7	.9399	.9520	2.572581	8	.9894	.9932	3.066351	9
15	12	.8686	.9016	2.215639	8	.9443	.9610	2.497632	8	.9842	.9902	2.928254	9
15	11	.8860	.9093	2.327573	7	.9442	.9582	2.450070	9	.9898	.9945	2.935794	8
15	10	.8945	.9134	2.288015	7	.9498	.9626	2.545869	8	.9846	.9912	2.800559	9
20	20	.8908	.9122	2.390453	6	.9487	.9533	2.651966	9	.9882	.9915	3.162272	9
20	19	.8959	.9013	2.399386	8	.9404	.9517	2.595445	9	.9882	.9902	3.092127	9
20	18	.8895	.9003	2.299963	8	.9498	.9556	2.599142	8	.9883	.9910	3.019933	9
20	17	.8994	.9112	2.429542	7	.9477	.9527	2.614971	9	.9893	.9916	3.121657	9
20	16	.8954	.9041	2.371707	8	.9427	.9512	2.624997	9	.9882	.9902	3.060679	9
20	15	.8857	.9049	2.330453	7	.9486	.9588	2.561737	9	.9884	.9908	3.014382	9
25	25	.8991	.9043	2.425355	7	.9482	.9526	2.649064	9	.9896	.9907	3.149702	9
25	24	.8981	.9086	2.444417	7	.9487	.9511	2.718331	9	.9890	.9901	3.112130	5
25	23	.8976	.9051	2.452055	7	.9486	.9515	2.686057	9	.9892	.9902	3.169415	9
25	22	.8924	.9008	2.446065	7	.9462	.9501	2.669665	9	.9894	.9909	3.129822	9
25	21	.8949	.9101	2.407484	7	.9456	.9533	2.633505	9	.9884	.9902	3.065758	9
25	20	.8969	.9112	2.449485	7	.9474	.9514	2.605796	9	.9896	.9910	3.134101	9
30	30	.8976	.9011	2.435991	8	.9490	.9510	2.738609	9	.9895	.9902	3.214797	5
30	29	.8939	.9016	2.479766	8	.9480	.9501	2.741594	9	.9900	.9908	3.213316	5
30	28	.8919	.9004	2.427485	8	.9481	.9514	2.694307	9	.9884	.9905	3.153223	9
30	27	.8967	.9010	2.473089	7	.9480	.9507	2.703231	9	.9900	.9907	3.187358	9
30	26	.8972	.9027	2.398616	8	.9471	.9500	2.647935	9	.9893	.9902	3.185199	9
30	25	.8948	.9017	2.482944	7	.9471	.9504	2.708011	9	.9898	.9906	3.155836	9
35	35	.8981	.9018	2.418961	8	.9487	.9504	2.700998	9	.9899	.9910	3.213693	9
35	34	.8973	.9001	2.439956	8	.9487	.9518	2.713050	9	.9896	.9901	3.235110	9
35	33	.8939	.9018	2.428476	8	.9497	.9517	2.669043	9	.9895	.9901	3.174093	9
35	32	.8984	.9017	2.433121	7	.9495	.9514	2.664516	9	.9897	.9904	3.221699	9
35	31	.8972	.9014	2.419553	8	.9493	.9515	2.688038	9	.9899	.9905	3.206271	9
35	30	.8982	.9014	2.418675	8	.9479	.9505	2.700287	9	.9897	.9906	3.182737	9
40	40	.8962	.9001	2.466618	8	.9494	.9524	2.727585	9	.9894	.9900	3.212875	9
40	39	.8980	.9008	2.440241	8	.9493	.9509	2.733963	9	.9897	.9901	3.273558	9
40	38	.8992	.9036	2.475984	8	.9489	.9503	2.723065	9	.9895	.9901	3.241824	9
40	37	.8978	.9002	2.422312	8	.9483	.9502	2.714061	9	.9899	.9907	3.260459	9
40	36	.8993	.9016	2.448949	8	.9495	.9510	2.736411	9	.9899	.9907	3.219251	9
40	35	.8984	.9010	2.424496	8	.9490	.9532	2.704213	9	.9895	.9900	3.192578	9
45	45	.8994	.9021	2.452761	8	.9495	.9515	2.751529	9	.9899	.9902	3.255902	9
45	44	.8992	.9016	2.472518	8	.9498	.9522	2.710952	9	.9897	.9900	3.279751	9
45	43	.8966	.9012	2.455377	8	.9491	.9502	2.744602	9	.9899	.9902	3.231605	5
45	42	.8972	.9031	2.470127	8	.9499	.9511	2.716575	9	.9897	.9901	3.262286	9
45	41	.8990	.9009	2.459192	8	.9496	.9507	2.741890	9	.9897	.9900	3.238484	9
45	40	.8995	.9016	2.462650	8	.9494	.9515	2.716326	9	.9897	.9901	3.259598	9
50	50	.8994	.9013	2.477696	8	.9490	.9502	2.735764	9	.9895	.9901	3.270847	9
50	49	.8953	.9005	2.480403	8	.9490	.9502	2.755113	9	.9896	.9901	3.268895	9
50	48	.8978	.9002	2.449985	8	.9496	.9507	2.746549	9	.9897	.9904	3.260541	9
50	47	.8971	.9006	2.467553	8	.9499	.9510	2.757227	9	.9900	.9903	3.304179	9
50	46	.8947	.9007	2.438465	8	.9495	.9505	2.744795	9	.9894	.9901	3.255123	9
50	45	.8989	.9006	2.491113	8	.9480	.9510	2.735209	9	.9897	.9903	3.264504	9
100	100	.8990	.9002	2.483005	9	.9495	.9502	2.775122	9	.9899	.9901	3.313039	9
500	500	.9000	.9001	2.522724	5	.9500	.9500	2.802861	9	.9900	.9900	3.365530	5

Appendix.

The purpose of this appendix is to summarize the algebraic results which we used in the preceding chapters. All proofs are straightforward verifications of the definitions. Hence, we give only some hints and leave the details to the reader. A more general approach including Eulerian polynomials can be found in [11]. The "Finite Operator Calculus" of G.-C. Rota, D. Kahaner and A. Odlyzko [14] is the fundament of the whole theory.

Let $\underset{\sim}{P}$ be the algebra of polynomials over a field K with characteristic zero. In our rank test applications K always equals $\mathbb{Z}$, for the order tests choose $K = \mathbb{R}$. We will deal with linear operators $\underset{\sim}{P} \to \underset{\sim}{P}$ only, and omit the word "linear" in the sequel. For all $a \in K$ the shift operator is denoted by E^a: $p(x) \mapsto p(x+a)$. An operator Q on $\underset{\sim}{P}$ is a delta operator, if

Q is shift invariant: $QE^a = E^aQ \quad \forall a \in K$, and

Qx is a non-zero constant.

The derivative operator D is a delta operator if $K = \mathbb{R}$, and the following properties show how Q generalizes D:

(A.1) $\quad Qa = 0$ for every constant a [14,p.687]

(A.2) $\quad \deg(Qp) = \deg(p)-1$ for each $p \in \underset{\sim}{P}$ with $\deg(p) \geq 1$ [14,p.687] .

Hence, the kernel of Q consists only of the constant polynomials.

A sequence of polynomials $(s_n)_{n \in \mathbb{N}_0}$ is a Sheffer sequence for Q, if

(A.3) s_0 is a non-zero constant

(A.4) $Qs_n = s_{n-1}$ for all $n \geq 1$.

We make the convention $s_n = 0$ if $n < 0$. For instance, $(x^n/n!)$ is a Sheffer sequence for D.

Lemma A.1: If (s_n) is a Sheffer sequence for Q then $\deg(s_n) = n$.

Proof: (A.1)-(A.4) ■

Lemma A.2: If (s_n) and (t_n) are both Sheffer sequences for Q with the property

$$s_n(\nu_n) = t_n(\nu_n)$$

for a given sequence (ν_n) in K, then the two sequences are equal.

Proof: Induction over n. Use $\ker(Q)$ = constant functions ■

(s_n) has roots in $\nu: \mathbb{N}_0 \to K$, say, if $s_n(\nu_n) = \delta_{0,n}$ for all $n \in \mathbb{N}_0$. The Sheffer sequence for Q with roots in 0 is called the basic sequence for Q and always denoted by (q_n). Obviously,

(A.5) $(x^n/n!)$ is the basic sequence for D .

It is easy to verify that

(A.6) $\left(\binom{x+n-1}{n}\right)_{n\in \mathbb{N}_0}$ is the basic sequence for $\nabla = I-E^{-1}$.

More examples can be found in [14].

Immediately from the shift-invariance follows: If (s_n) is a Sheffer sequence for Q with roots in ν, then $(E^a s_n)$ is a Sheffer sequence for Q with roots in $\nu-a$.

Deeper than all the other results in this appendix is the following

Lemma A.3: If $\nu(n) = an+b$ $(a,b\in K)$, then

$$s_n(x) := (x-an-b)(x-b)^{-1}q_n(x-b)$$

defines the Sheffer sequence for Q with roots in ν.
(For $n = 0$ we have to define $\frac{0}{0} = 1$.)

Proof: See [14, p. 702] ∎

Now we come to a representation theorem for Sheffer sequences with roots in

$$\nu(i) := \begin{cases} \varphi(i) & \forall 0 \leq i \leq L \\ ci+d & \forall i > L, \end{cases}$$

where $L \in \mathbb{N}_0$; $c,d\in K$ and $\varphi: \mathbb{N}_0 \to R$ arbitrary.

Theorem A.1: If (s_n) is the Sheffer sequence for Q with roots in ν as above, then

(A.7) $$s_n(x) = \sum_{i=0}^{L} s_i(ci+d)(x-cn-d)(x-ci-d)^{-1}q_{n-i}(x-ci-d) \quad \forall n \in \mathbb{N}_0 .$$

Proof: Check recurrence and side conditions, using lemma A.3 ∎

Corollary A.1: (Binomial Theorem). If (s_n) is the Sheffer- and (q_n) the basic sequence for Q, then

$$s_n(x+y) = \sum_{i=0}^{n} s_i(y)q_{n-i}(x) \quad \forall n \in \mathbb{N}_0 .$$

Proof: Choose $c = 0$, $d = y$ and $L = \infty$ in (A.7) ∎

Avoiding alternating summation in (A.7), it may be sometimes preferable to use the "outside method" (a term, introduced by J.L. Hodges (1957)):

$$\text{(A.8)} \qquad s_n(x) = r_n(x) - \sum_{i=L+1}^{n} r_i(ci+d)(x-cn-d)(x-ci-d)^{-1}q_{n-i}(x-ci-d)$$

where (r_n) is the Sheffer sequence for Q with roots in φ (follows by summation over all $i = 0,\ldots,n$ in (A.7)).

Repeated use of (A.7) yields a representation of the Sheffer sequence (s_n) for Q with roots in the piecewise affine function

$$\nu(i) := ia_j+b_j \quad \forall L_j < i \leq L_{j+1} ,$$

where $-1 = L_0 < L_1 < \cdots$, each L_j integer, and $a_j,b_j \in K$ for all $j \in \mathbb{N}_0$. Then for all $L_j < n \leq L_{j+1}$

$$\text{(A.9)} \qquad s_n(x) = \sum_{k_j=0}^{L_j} \cdots \sum_{k_1=0}^{L_1} p_j(x)p_{j-1}(\nu_j(k_j))\cdot\ldots\cdot p_0(\nu_1(k_1)) ,$$

if

$$p_i(x) = \frac{x-\nu_i(k_{i+1})}{x-\nu_i(k_i)} q_{k_{i+1}-k_i}(x-\nu_i(k_i)) ,$$

where $k_0 := 0$ and $k_{j+1} := n$. Because of its importance we explicitly write down the special case of (A.9) where

$$\nu(i) = \begin{cases} ia+b & \forall i = 0,\ldots,L \\ ic+d & \forall i > L . \end{cases}$$

Then

$$\text{(A.10)} \qquad s_n(x) = \sum_{i=0}^{L} \frac{i(c-a)+d-b}{ic+d-b} q_i(ci+d-b) \frac{x-cn-d}{x-ci-d} q_{n-i}(x-ic-d) .$$

For $n \leq L$, the r.h.s. equals $(x-an-b)(x-b)^{-1}q_n(x-b)$ by lemma A.3.

Now we assume that K is completely ordered. Let $\mu: \mathbb{N}_0 \to K$ be a non decreasing function and $(t_{n,i})_{n,i \in \mathbb{N}_0}$ be a double sequence in $\underset{\sim}{P}$ with the properties

$$\text{(A.11)} \qquad \begin{aligned} & t_{n,i}(\mu_i) = t_{n,i+1}(\mu_i) && \forall 0 \leq i \leq r(n) := \min\{m \in \mathbb{N}_0 \,|\, \mu(m)=\mu(n)\}, \\ & t_{n,i} = 0 && \forall i > r(n) . \end{aligned}$$

Define an associated sequence (f_n) to $(t_{n,i})$ by

$$\text{(A.12)} \qquad f_n(x) := t_{n,i}(x) \quad \forall \mu_{i-1} < x \leq \mu_i \quad (\mu_{-1} := -\infty) .$$

We call (f_n) a μ-Sheffer sequence, if $\left(t_{m+n,r(m)}\right)_{n \in \mathbb{N}_0}$ is a Sheffer

sequence for all $m \in \mathbb{N}_0$. From corollary A.1 we get a first representation of $f_n(x)$:

$$\text{(A.13)} \quad f_n(x) = \sum_{k=i}^{n} f_k(y) q_{n-k}(x-y) \quad \text{if} \quad x,y \in [\mu_{i-1}, \mu_i] \ .$$

If (f_n) has roots in ν, i.e.

$$\text{(A.14)} \qquad f_n(\nu_n) = \delta_{0,n} \quad \forall n \in \mathbb{N}_0 \ ,$$

then any value $f_n(z)$ can be computed from (A.13) by stepping through all the intervals $[\mu_j, \mu_{j+1}]$ until z is enclosed. We give only a brief description of this trivial algorithm:

<u>Algorithm A.1</u>. Assume $f_{r(j)}(\mu_j), \ldots, f_i(\mu_j)$ are already computed such that $j \leq n$ and $\nu_{i+1} > \mu_j$

a) If $\nu_{i+1} < \mu_{j+1}$ then define $x := \nu_{i+1}$, $y := \mu_j$, and compute $f_{r(j)}(\nu_{i+1}), \ldots, f_i(\nu_{i+1})$ from (A.13). Of course, $f_{i+1}(\nu_{i+1}) = 0$. Therefore, the i-index increased by one, and it increases again if ν_{i+2} lies also in the same interval (define $x = \nu_{i+2}$ and $y = \nu_{i+1}$). Finally a k is reached such that $\mu(j) < \nu_k < \mu_{j+1} < \nu_{k+1}$ (the case $\nu_k = \mu_{j+1}$ is left to the reader). Then choose $x := \mu_{j+1}$, $y := \nu_k$, and compute $f_{r(j)}(\mu_{j+1}, \ldots, f_k(\mu_{j+1})$ from (A.13). Now we are in the same situation as in the beginning.

b) If $\nu_{i+1} > \mu_{j+1} > \mu_j$ then define $x := \mu_{j+1}$, $y := \mu_j$, and compute $f_{r(j)}(\mu_{j+1}, \ldots, f_k(\mu_{j+1})$ from (A.13). Again, we are in the same situation as in the beginning.

c) If $\mu_j = \mu_{j+1}$ increase j by one.

In special cases this algorithm can be simplified.

A one dimensional recursion can be obtained from

Theorem A.2: Let (f_n) be a μ-Sheffer sequence for Q (with basic sequence (q_n)). If (f_n) is associated to $(t_{n,i})$ then

$$\text{(A.15)} \qquad t_{n,i}(x) = \sum_{k=i}^{n} f_k(\mu_k) q_{n-k}(x-\mu_k) \quad \text{for all } n \in \mathbb{N}_0 \text{ and } i = 0,\dots,n .$$

Proof: Verify side conditions (A.11).

See [26, theorem 4.1] for a general version of this theorem. The announced one dimensional recursion follows, when we write (A.15) as

$$\text{(A.16)} \qquad f_n(x) = \Sigma' f_k(\mu_k) q_{n-k}(x-\mu_k) \quad \text{for all } n \in \mathbb{N}_0 ,$$

where the summation runs over all k such that $\mu_k > x$. Thus, a system of equations for the unknown $f_k(\mu_k)$ is obtained, if only one value $f_n(\nu_n)$ with $\nu_n \le \mu_n$ is known for each n. By Cramer's rule, $f_n(\mu_n)$ can be expressed as a determinant:

Corollary A.2: If (f_n) is a μ-Sheffer- and (q_n) the basic sequence for Q, then

$$f_n(\mu_n) = \det(\alpha_{i,j})_{i,j=1,\dots,n+1} , \quad \text{where}$$

$$\alpha_{i,j} = \begin{cases} q_{i-j}((\nu_{i-1}-\mu_{j-1})_-) & \forall j = 1,\dots,n \\ f_{i-1}(\nu_{i-1}) & \text{if } j = n+1 , \end{cases}$$

for any $\nu \le \mu$. If, in addition, (f_n) has roots in ν, then

$$\text{(A.17)} \qquad f_n(\mu_n) = (-1)^n \det(q_{i+1-j}((\nu_i-\mu_{j-1})_-))_{i,j=1,\dots,n} .$$

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A Kolmogorov-Smirnov statistic with the weight factor $F(x)(1-F(x))$ is called "variance-weighted". We give tables of the exact significance points for various one and two sample cases.

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